Stat 400 **MIDTERM 2** **Nov. 21 , 2002**

J.Millson

1. Suppose X and Y are random variables defined on the same sample space with the following joint probability mass function.

X \ Y	0	1
0	0	1/4
1	1/4	1/2

(a) Compute the probability mass functions of the random variables X and Y.
(b) Are X and Y independent?
(c) Compute the probability mass function of the random variable $Z = X + Y$.
(d) Compute $Cov(X, Y)$.
(e) Compute the correlation $\rho_{X,Y}$.

(25 points)

2. Suppose that X and Y are independent random variables defined on the same sample space. Suppose both X and Y have geometric distribution with parameter p. How is the sum $Z = X + Y$ distributed?

(10 points)

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3. Let the pair X and Y have the joint probability mass function of Problem 2, that is $p_{X,Y}$ is given by the matrix A

X \ Y	0	1
0	0	1/4
1	1/4	1/2

(i) Compute the four conditional probabilities
$P(X = 0|Y = 0), P(X = 0|Y = 1), P(X = 1|Y = 0), P(X = 1|Y = 1)$.

(ii) Arrange the four conditional probabilities you just computed in the 2 by 2 matrix B whose entry in the $(x, y)-th$ position is the conditional probability $P(X = x|Y = y)$.

(Recall that the conditional probability $P(A|B)$ of an event A given another event B is given by the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$. You have to interpret each entry in the matrix A given in the beginning of the problem as probability of an intersection of the two events $(X = x)$ and $(Y = y)$. Then you can pass from the entries of the matrix A to the entries of the matrix B.)

(10 points)

4. Suppose X has uniform distribution on $[0, 1]$. Let $Y = \sqrt{X}$. Find the density function $f_Y(y)$ of Y using the "Engineer's Way".

(5 points)