Stat 400   STAT 400 MIDTERM 2   April 29, 2010
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1. Suppose X and Y are random variables defined on the same sample space with the following joint probability mass function.

x \ y	-1	0	1
-1	0	1/4	0
0	1/4	0	1/4
1	0	1/4	0

(a) Compute the probability mass functions of the random variables X and Y.
(b) Are X and Y independent?
(c) Compute the probability mass function of the random variable $W = X + Y$.
(d) Compute the probability mass function of the random variable $W = Max(X, Y)$.
(e) Compute the covariance $Cov(X, Y)$ of X and Y.
(25 points)

2. Suppose that $x_1, x_2, \cdots, x_n$ is a sample from the space of a random variable X with density function

$$f(x) = \begin{cases} \frac{x^\theta}{\theta+1}, & 0 \le x \le 1 \\ 0, & otherwise. \end{cases}$$

Find the maximum likelihood estimator for the parameter θ.
(10 points)

3. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a population with mean μ. Show that the sample mean $\overline{X}$ is an unbiased estimator of the population mean μ (part of the problem is knowing what this statement means).
(10 points)

Turn the page.

4. John, Dave and Bob have a disagreement over a woman and decide to settle it with a three-cornered pistol duel. Of the three, John is the worst shot and hits his target only 30% of the time, Dave hits the target 50% of the time and Bob never misses. John shoots first then Dave, then Bob. Each shooter has an unlimited supply of bullets and shoots at the person most dangerous to him (the better shot). In what follows we will assume that nobody gets wounded, either he is untouched or killed.

(a) What is the probability that if John shoots and kills Dave on his first shot he will get a second shot?

(b) What is the probability that if John shoots and kills Bob on his first shot he will get another shot?

(c) What is the probability that if John doesn't hit anybody on his first shot he will get a second shot?

(d) In the light of the answers to (a),(b) and (c) what should John do with his first shot?

(5 points, 2 points for (c) and 1 point for each of the rest)