Stat 400 **PRACTICE FINAL EXAM**

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1. Let X be a discrete random variable with probability mass function p given by

$$p(1) = 1/3, \ p(2) = 1/3, \ p(3) = 1/3.$$

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(a) Find $E(X)$.
(b) Find $E(X^2)$.
(c) Find $V(X)$.
(d) Find $F(x)$, the cumulative distribution function of X.
(e) Make the change of variable $Y = X - 1$. Find the probability mass function of the new random variable Y.

(20 points)

2. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} 2(1-x), \ 0 \leq x \leq 1, \\ 0, otherwise. \end{cases}$$

(a) Find E(X).
(b) Find V(X).
(c) Find $F(x)$, the cumulative distribution function of X.
(d) Find the median of X.
(e) Find the 75-th percentile of X

(20 points)

3. Two couples (Jack and Jill and Dick and Jane) go to the movies and are seated randomly in four adjacent seats. What is the probability some husband sits beside his wife?

(10 points)

4. Suppose X and Y are random variables defined on the same sample space with the following joint probability mass function.

X \ Y	0	1
0	0	1/4
1	1/4	1/2

(a) Compute the probability mass functions of the random variables X and Y.
(b) Are X and Y independent?
(c) Compute the probability mass function of the random variable $Z = X + Y$.
(d) Compute $Cov(X, Y)$.
(e) Compute the correlation $\rho_{X,Y}$.

(20 points)

5. A sample of 26 offshore oil workers took part in a simulated escape exercise. The resulting 26 escape times were recorded with a sample mean of 24.36 and a sample standard deviation of 370.69.

(i) Calculate a 95% lower-tailed confidence interval (upper confidence bound) for population mean escape time.

(ii) Calculate a 95% lower-tailed prediction interval (upper prediction bound) for a single additional worker.

(20 points)

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Let $\overline{X} = \frac{X_1+\cdots+X_n}{n}$ be the sample mean for the first n observations and $S^2 = \frac{\Sigma_{i=1}^n (X_1-\overline{X})^2}{n-1}$ be the sample variance for the first n observations. Assume the theorem that $T = \frac{\overline{X}-\mu}{\frac{S}{\sqrt{n}}}$ has t distribution with $n-1$ degrees of freedom. Prove that the random interval $(\overline{X} - t_{\alpha,n-1} \frac{1}{\sqrt{n}} S, \infty)$ is a $100(1-\alpha)\%$ confidence interval for μ.

(10 points)