

1. Let  $X$  be a discrete random variable with probability mass function  $p$  given by

$$p(0) = 1/6, p(1) = 1/3, p(2) = 1/3, p(3) = 1/6$$

- (a) Find  $E(X)$ .
- (b) Find  $E(X^2)$ .
- (c) Find  $V(X)$ .
- (d) Find  $F(x)$ , the cumulative distribution function of  $X$ .

(16 points)

2. Let  $X$  be a continuous random variable with probability density function

$$f(x) = \begin{cases} 4x^3, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find  $E(X)$ .
- (b) Find  $V(X)$ .
- (c) Find  $F(x)$ , the cumulative distribution function of  $X$ .
- (d) Find  $\tilde{\mu}$ , the median of  $X$ .

(16 points)

3. Find the probability of 2 triples and 2 pairs (eg KKKQQQ2255) in a 10 card poker hand.

(9 points)

4. A population starts with one member at time  $t = 1$ . It either divides in two with probability  $p$  or dies with probability  $1 - p$ . If it divides, then both of its children behave independently with the same two alternatives at time  $t = 2$ . Let  $X_t$ ,  $t = 1, 2, \dots$  be the number of members of the population at the time  $t$ .

1. Find the probability distribution of the random variable  $X_2$  (this means: describe the possible values of  $X_2$  and their probabilities. Note that  $X_1$  takes the value 1 with probability 1).
2. Find the probability distribution of the random variable  $X_3$ .

(9 points)