Stat 400 STAT 400 PRACTICE MIDTERM 2 April 28, 2010
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1. Suppose X and Y are random variables defined on the same sample space with the following joint probability mass function.

x \ y	0	1	2
0	1/16	1/16	1/8
1	1/8	1/4	1/8
2	1/16	1/16	1/8

(a) Compute the probability mass functions of the random variables X and Y.
(b) Are X and Y independent?
(c) Compute the probability mass function of the random variable $W = X + Y$.
(d) Compute the covariance $Cov(X, Y)$ of X and Y.
(e) Compute the correlation $\rho_{X,Y}$ of X and Y.
(15 points)

2. In what follows X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{\theta^2} x \exp -x/\theta, x \geq 0, \\ 0, x < 0. \end{cases}$$

To do part (b) below you will need

$$E(X) = 2\theta.$$

Let $x_1, x_2, \ldots, x_n$ be a sample from the space of X.

(a) Compute the maximum likelihood estimator of θ.

(b) Prove that the estimator from (a) is unbiased.
(15 points)

3. Prove that the sample mean $\overline{X}$ is an unbiased estimator of the population mean μ. (10 points).

4. Suppose the distribution of the time X (in hours) spent by students on their Stat 400 homework per week has a continuous distribution with mean 50 and variance 100. Assume that X can be approximated by a normal distribution. Find an approximation for the probability that a randomly selected student spends more than 60 hours per week on his or her Stat 400 homework.

(10 points).