

# Stat 400

## Solutions to Midterm I (Spring 2010)

(a)

$x$	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$(b) \quad E(X) = (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) \\ + (4)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{6}\right)$$

$$= \left(\frac{1}{6}\right)(1+2+3+4+5+6)$$

$$= \left(\frac{1}{6}\right)(21) = 2\frac{1}{6} = \frac{7}{2}$$

$$(c) \quad E(X^2) = (1^2)\left(\frac{1}{6}\right) + (2^2)\left(\frac{1}{6}\right) + (3^2)\left(\frac{1}{6}\right) \\ + (4^2)\left(\frac{1}{6}\right) + (5^2)\left(\frac{1}{6}\right) + (6^2)\left(\frac{1}{6}\right)$$

$$= \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2)$$

$$= \frac{91}{6}$$

(f) Just shift all the values of  $X$  with their probabilities one unit to the left.

y	0	1	2	3	4	5
$P(Y=y)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

2.  
(a)  $E(X) = \int_0^1 x \cdot 2(1-x) dx$

$$= 2 \left( \int_0^1 x dx - \int_0^1 x^2 dx \right)$$

$$= 2 \left( \frac{1}{2} - \frac{1}{3} \right) = 2 \left( \frac{1}{6} \right)$$

$$= \frac{1}{3}$$

So  $V(X) = \frac{1}{6} - \left(\frac{1}{3}\right)^2$

(b)  $E(X^2) = \int_0^1 x^2 \cdot 2(1-x) dx$

$$= 2 \left( \int_0^1 x^2 dx - \int_0^1 x^3 dx \right)$$

$$= 2 \left( \frac{1}{3} - \frac{1}{4} \right) = 2 \left( \frac{1}{12} \right) = \frac{1}{6}$$

(c) If  $x \leq 0$  then  $F(x) = 0$  4

If  $0 < x < 1$  then

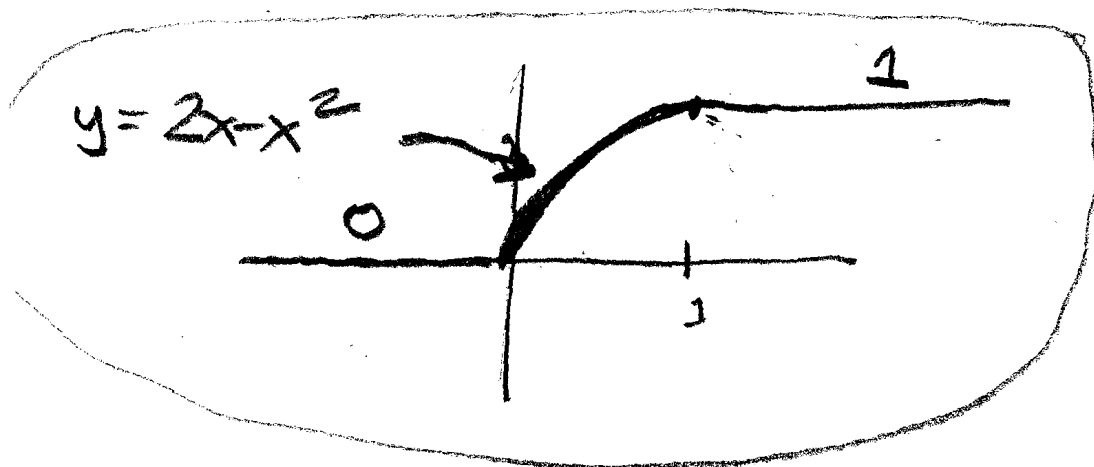
$$F(x) = \int_0^x 2(1-t) dt = 2 \left( \int_0^x dt - \int_0^x t dt \right)$$

$$= 2 \left[ (t) \Big|_{t=0}^{t=x} - \left( \frac{t^2}{2} \right) \Big|_{t=0}^{t=x} \right]$$

$$= 2 \left[ x - \frac{x^2}{2} \right] = 2x - x^2$$

So

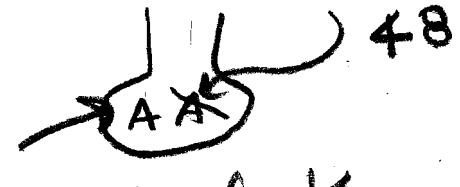
$$F(x) = \begin{cases} 0, & x < 0 \\ 2x - x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$



← not required

2. This is a special case of the hypergeometric distribution.

Imagine a container with 50 cards (because two have been dealt) 2 aces (because 2 aces " " " ) 48 non-aces.

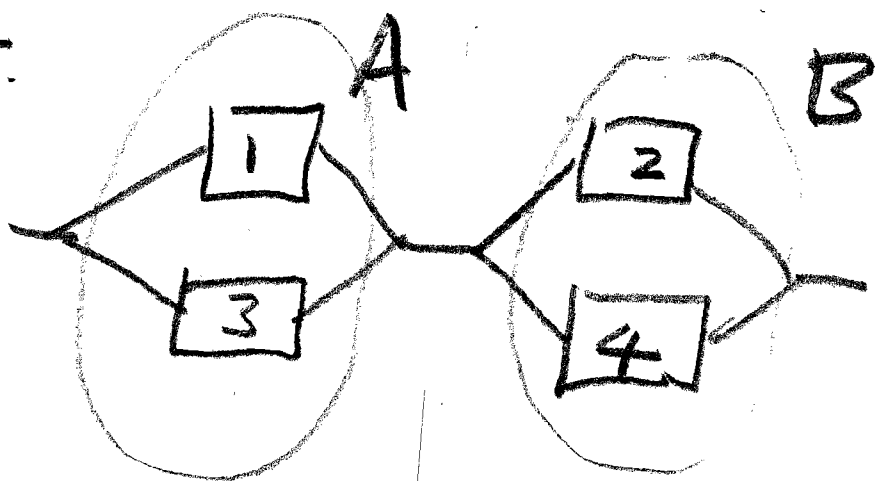


We pick 3 cards out of the container - so  $\binom{50}{3}$  ways.

We want the number of ways to pick 2 aces so  $\binom{2}{2}$  ways AND for each of those ways to pick one non-ace so  $\binom{48}{1} = 48$  ways. So

$$P = \frac{\binom{2}{2} \binom{48}{1}}{\binom{50}{3}} = \frac{48}{\binom{50}{3}}$$

4.



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Lump 1 and 3 into A  
and 2 and 4 into B

Define events

$S =$  system works

$A =$  A works (bad notation  
but it's OK)

$B =$  B works

So (here is the point)

$S = A \cap B$

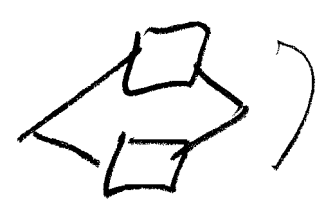
(system works  $\Leftrightarrow$  BOTH A and B work)

Since A and B don't  
 have a component in common  
 and the components function  
 independently

↖ not required

A and B are independent and

$$P(S) = P(A)P(B)$$

Since A and B are identical  
 systems (both are )

we have

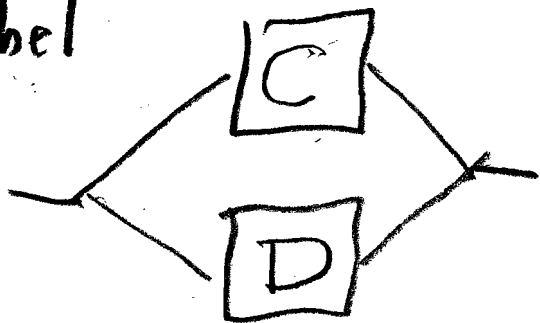
$$P(A) = P(B)$$

so

$$P(S) = P(A)^2$$

It remains to compute  $P(A)$  ?

Relabel



Define events

$C = C$  works

$D = D$  works

$A = C \cup D$  ( $A$  works  $\Leftrightarrow$  EITHER  $C$  or  $D$  works)

so

$$P(A) = P(C) + P(D) - P(C \cap D)$$

Now  $C$  and  $D$  are independent so

$$P(C \cap D) = P(C)P(D) = p^2$$

so  $P(A) = 2p - p^2$  and

$$P(S) = (2p - p^2)^2$$