The basic probability distributions

August 25, 2005

1 The discrete distributions

1. Bernoulli with parameter p, text pg. 105

$$\begin{aligned} p(x) &= p^x(1-p)^{1-x},\ x=0,1 \\ M(t) &= 1-p+pe^t \\ E(X) &= p \\ V(X) &= pq \end{aligned}$$

2. Binomial with parameters n and p, text pg. 120-126

$$\begin{aligned} p(x) &= \binom{n}{x} p^x(1-p)^{n-x},\ x=0,1,\cdots,n \\ M(t) &= (1-p+pe^t)^n \\ E(X) &= np \\ V(X) &= npq \end{aligned}$$

3. Hypereometric with parameters N, M and n,text pg. 129-131

$$\begin{aligned} p(x) &= \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}},\ x \le n, x \le M \text{ and } max(0, n-N+M) \le x \le min(n, M) \\ E(X) &= \frac{nM}{N} \\ V(X) &= \frac{N-n}{N-1} \cdot n \cdot \frac{M}{N} \cdot (1-\frac{M}{N}) \end{aligned}$$

4. Negative binomial with parameters r and p, text pg. 132

$$
\begin{aligned}
p(x) &= \binom{x+r-1}{r1} p^r(1-p)^x,\ x = 0, 1, 2, \cdots \\
M(t) &= \frac{p^r}{(1-(1-p)e^t)^r} \\
E(X) &= \frac{r(1-p)}{p} \\
V(X) &= \frac{r(1-p)}{p^2}
\end{aligned}
$$

5. Geometric with parameter p, text pg. 133

$$
\begin{aligned}
p(x) &= p(1-p)^x,\ x = 0, 1, 2, \cdots \\
M(t) &= \frac{p}{1-(1-p)e^t} \\
E(X) &= \frac{1-p}{p} \\
V(X) &= \frac{1-p}{p^2}
\end{aligned}
$$

Remark The geometric distribution is the special case of the negative binomial distribution with

$$r = 1 \text{ and } p = p.$$

6. Poisson with parameter λ, text pg. 135-138

$$
\begin{aligned}
p(x) &= \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0, 1, 2, \cdots \\
M(t) &= e^{\lambda(e^t-1)} \\
E(X) &= \lambda \\
V(X) &= \lambda
\end{aligned}
$$

2 The continuous distributions

1. Uniform distribution on [A,B], text pg. 148

$$f(x) = \begin{cases} \frac{1}{B-A}, & A \le x \le B \\ 0, & \text{otherwise} \end{cases}$$
$$M(t) = \frac{e^{Bt} - e^{At}}{t(B-A)}$$
$$E(X) = \frac{A+B}{2}$$
$$V(X) = \frac{(B-A)^2}{12}$$

2. Gamma distribution with parameters α and β, text pg. 175-176

$$f(x) = \begin{cases} 0, x < 0 \\ \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, & x \ge 0 \end{cases}$$
$$M(t) = \frac{1}{(1-\beta t)^\alpha}$$
$$E(X) = \alpha\beta$$
$$V(X) = \alpha\beta^2$$

3. Exponential with parameter λ, text pg. 177

$$f(x) = \begin{cases} 0, & x < 0 \\ \lambda e^{-\lambda x}, & x \ge 0 \end{cases}$$
$$M(t) = \frac{1}{1-\frac{t}{\lambda}}$$
$$E(X) = \frac{1}{\lambda}$$
$$V(X) = \frac{1}{\lambda^2}$$

Remark The exponential distribution is the special case of the gamma distribution with

$$\alpha = 1 \text{ and } \beta = \frac{1}{\lambda}.$$

4. The chi-squared distribution with parameter ν , text pg. 179 We will abbreviate this to $\chi^2(\nu)$.

$$f(x) = \begin{cases} 0, \ x < 0 \\ \frac{x^{\frac{\nu}{2}-1}e^{-\frac{x}{2}}}{2^{\frac{\nu}{2}}\Gamma(\frac{\nu}{2})}, \ x \geq 0 \end{cases}$$
$$M(t) = \frac{1}{(1-2t)^{\frac{\nu}{2}}}$$
$$E(X) = \nu$$
$$V(X) = 2\nu$$

Remark The chi-squared distribution is the special case of the gamma distribution with

$$\alpha = \frac{\nu}{2} \text{ and } \beta = 2.$$

5. The normal distribution with parameters μ and σ^2, text pg. 165-168

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \ \infty < x < \infty$$
$$M(t) = \ e^{\mu t + \frac{\sigma^2 t^2}{2}}$$
$$E(X) = \ \mu$$
$$V(X) = \ \sigma^2$$

5. The standard normal, text pg. 161-165

$$f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(x)^2}{2}}, \ \infty < x < \infty$$
$$M(t) = \ e^{\frac{t^2}{2}}$$
$$E(X) = \ 0$$
$$V(X) = \ 1$$

Remark The standard normal distribution is the special case of the normal distribution with

$$\mu = 0 \text{ and } \sigma^2 = 1.$$