HW2

Solutions to the Extra Problems

1. (i) Pick an unordered triple of kinds $\binom{13}{3}$
   (ii) Pick three from the 1st kind $\binom{4}{3}$
   (iii) " " " " 2nd kind $\binom{4}{3}$
   (iv) " " " " 3rd kind $\binom{4}{3}$

$$P(3 \text{ triples}) = \frac{\binom{13}{3}\binom{4}{3}\binom{4}{3}\binom{4}{3}}{\binom{52}{9}}$$

2. Call the couples Jack and Jill, Dick and Jane, Bonnie and Clyde.

Let A = Jack and Jill sit together
B = Dick and Jane sit together
C = Bonnie and Clyde sit together
S = Some couple sits together

Hence

$$S = A \cup B \cup C$$

Note # arrangement of the six people $= 6!$

By the Principle of Inclusion and Exclusion

$$\#(S) = \#(A \cup B \cup C)$$

$$= \#(A) + \#(B) + \#(C)$$

$$- \#(A \cap B) - \#(A \cap C) - \#(B \cap C)$$

$$- \#(A \cap B \cap C) \qquad (*)$$

To compute $\#(A)$ think of Jack and Jill as one object — 5 objects

$$\underbrace{\text{Jack \& Jill} \quad D \quad J \quad B \quad C}$$

$$\#(A) = \underbrace{5!}_{\text{arrangements of the 5 objects}} \times 2$$

you can interchange Jack and Jill

Since $\#(A) = \#(B) = \#(C)$

They are all equal to $2 \times 5! = \underline{240}$

To compute $\#(A\cap B)$ think of Jack and Jill as one unit and Dick and Jane as one unit

4 objects

Jack & Jill   Dick & Jane   B   C

$\#(A\cap B) = 4! \times 2^2$

you can interchange Jack and Jill, and Dick and Jane

Since $\#(A\cap B) = \#(A\cap C) = \#(B\cap C)$ they are all equal to $4\times 4! = \underline{96}$

To compute $\#(A\cap B\cap C)$ we consider each of the couples as a single unit

3 objects

Jack & Jill   Dick & Jane   Bonnie & Clyde

$(A\cap B\cap C) = 3! \times 2^3$

you can interchange J & J, D & Jane, B & C.

$= \underline{48}$

Plugging in to (*) we get

$$\#(S) = 240 + 240 + 240 - 96 - 96 - 96 + 48$$

$$= 720 - 288 + 48 = 480$$

$$P(S) = \frac{480}{6!} = \frac{480}{720} = \frac{2}{3}$$

3. We want $P(\text{Red}_{\text{down}} \mid \text{Red}_{\text{up}})$

By the definition of conditional probability we have

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Note $\text{Red}_{\text{up}} \cap \text{Red}_{\text{down}} = \text{card } A$

(where $A = RR$, $B = BR$, $C = BB$)

so $$P(A \mid B) = \frac{P(\text{card } A)}{P(\text{Red}_{\text{down}})} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

There is a lot more to say about this problem.

4 total # of students = 500 + 300 + 200
= 1000

# of students who used Mean = 500

" " " " Median = 300

" " " " " = 200

so $P(\text{Mean}) = \frac{1}{2}$, $P(\text{Median}) = \frac{3}{10}$, $P(\text{Mode}) = \frac{1}{5}$

Let Sat denote the probability that a randomly selected student was satisfied with the test he/she got.

We want $P(\text{Mean} \mid \text{Sat})$

By the version of Bayes' Theorem in the text (Formula 2-6 with $A_1 = \text{Mean}$, $A_2 = \text{Median}$, $A_3 = \text{Mode}$ so $k = 3$)

$$P(\text{Mean} \mid \text{Sat}) = \frac{P(\text{Sat}/\text{Mean})\, P(\text{Mean})}{P(\text{Sat}/\text{Mean}) P(\text{Mean}) + P(\text{Sat}/\text{Median}) P(\text{Median}) + P(\text{Sat}/\text{Mode}) P(\text{M.}}$$

$$= \frac{\left(\frac{2}{5}\right)\left(\frac{1}{2}\right)}{\left(\frac{2}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{5}\right)} = \frac{20}{51}$$

An easier way

Use the formula (BT 2) from class

$$P(\text{Mean} \mid \text{Sat}) = \frac{P(\text{Mean})}{P(\text{Sat})} \underbrace{P(\text{Sat} \mid \text{Mean})}_{}$$

old numerator

Note $\#(\text{Sat}) = 200 + 150 + 160 = 510$

so $P(\text{Sat}) = \frac{510}{1000}$

Hence

$$P(\text{Mean} \mid \text{Sat}) = \frac{\left(\frac{1}{2}\right)}{\left(\frac{510}{1000}\right)} \left(\frac{2}{5}\right) = \frac{\left(\frac{2}{10}\right)}{\left(\frac{510}{1000}\right)}$$

$$= \frac{\frac{200}{1000}}{\frac{510}{1000}} = \frac{20}{51}$$

5. Let $X = \#$ of girls in the family

so $P(X=2) = \frac{1}{4}$, $P(X=1) = \frac{1}{2}$, $P(X=0) = \frac{1}{4}$

We want

$$P(X=2 \mid X \geq 1) = \frac{P(\underbrace{(X=2) \cap (X \geq 1)}_{(X=2)})}{P(X \geq 1)}$$

($A \cap B = A$ if $A \subset B$)

$$= \frac{P(X=2)}{P(X \geq 1)} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)} = \frac{1}{3}$$