

HW2

Solutions to the Extra Problems

1. (i) Pick an unordered triple of kinds $\binom{13}{3}$
(ii) Pick three from the 1st kind $\binom{4}{3}$
(iii) " " " " 2nd kind $\binom{4}{3}$
(iv) " " " " 3rd kind $\binom{4}{3}$

$$P(3 \text{ triples}) = \frac{\binom{13}{3} \binom{4}{3} \binom{4}{3} \binom{4}{3}}{\binom{52}{9}}$$

2. Call the couples Jack and Jill, Dick and Jane, Bonnie and Clyde.

Let $A =$ Jack and Jill sit together
 $B =$ Dick and Jane sit together
 $C =$ Bonnie and Clyde sit together
We $S =$ Some couple sits together

Hence

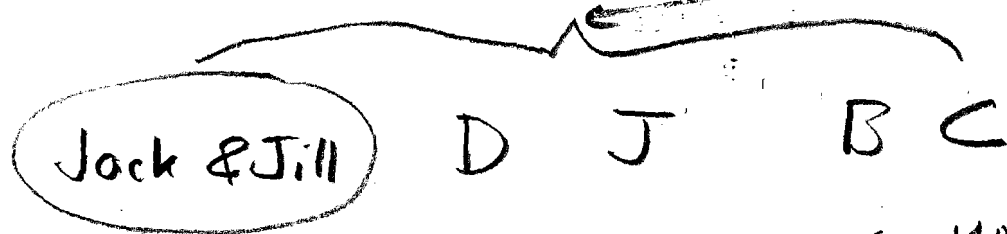
$$S = A \cup B \cup C$$

Note # arrangement of the six people = $6!$

By the Principle of Inclusion and Exclusion ²

$$\begin{aligned}\#(S) &= \#(A \cup B \cup C) \\ &= \#(A) + \#(B) + \#(C) \\ &\quad - \#(A \cap B) - \#(A \cap C) - \#(B \cap C) \\ &\quad + \#(A \cap B \cap C) \quad (*)\end{aligned}$$

To compute $\#(A)$ think of Jack and Jill as one object — 5 objects

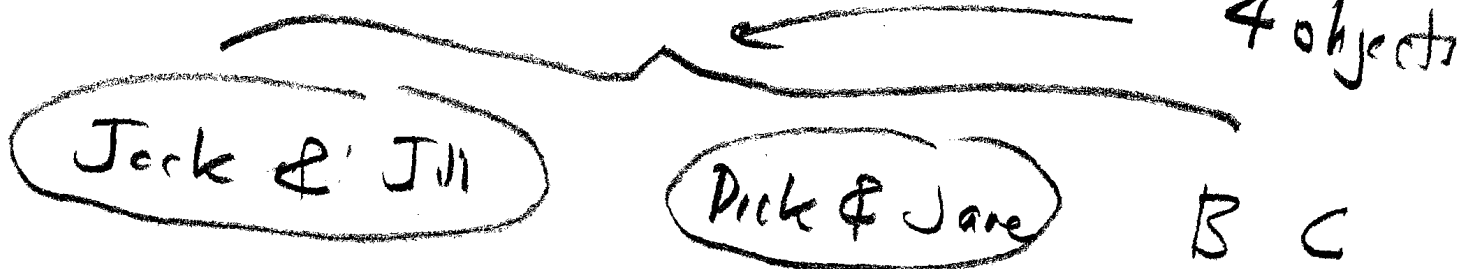


$$\#(A) = \underbrace{5!}_{\text{arrangements of the 5 objects}} \times \underbrace{2}_{\text{you can interchange Jack and Jill}}$$

Since $\#(A) = \#(B) = \#(C)$

They are all equal to $2 \times 5! = \underline{240}$

To compute $\#(A \cap B)$ think of Jack and Jill as one unit and Dick and Jane as one unit



$$\#(A \cap B) = 4! \times 2^2$$

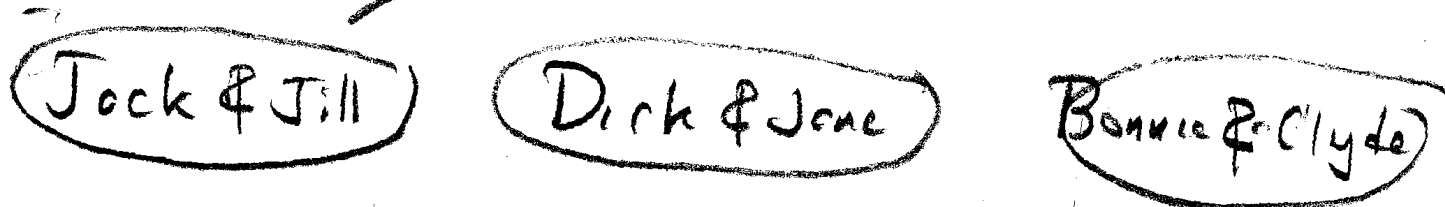
you can interchange Jack and Jill, and Dick and Jane

Since $\#(A \cap B) = \#(A \cap C) = \#(B \cap C)$

they are all equal to $4 \times 4! = \underline{96}$

To compute $\#(A \cap B \cap C)$

we consider each of the couples and single unit



$$\#(A \cap B \cap C) = 3! \times 2^3 = \underline{48}$$

you can interchange J & J, D & J and B & C.

Plugging in to (x) we get

$$\begin{aligned}\#(S) &= 240 + 240 + 240 \\ &\quad - 96 - 96 - 96 \\ &\quad + 48 \\ &= 720 - 288 + 48 = 480\end{aligned}$$

$$P(S) = \frac{480}{6!} = \frac{480}{720} = \frac{2}{3}$$

3. We want $P(\text{Red}_{\text{down}} | \text{Red}_{\text{up}})$

By the definition of conditional probability we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Note $\text{Red}_{\text{up}} \cap \text{Red}_{\text{down}} = \text{card } A$

(where $A = RR$, $B = BR$, $C = BB$)

so

$$P(A|B) = \frac{P(\text{card } A)}{P(\text{Red}_{\text{down}})} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

There is a lot more to say about this problem.

$$4. \quad \text{total \# of students} = 500 + 300 + 200 = 1000$$

of students who used Mean = 500

" " " Median = 300

so # " " " Mode = 200

$$P(\text{Mean}) = \frac{1}{2}, \quad P(\text{Median}) = \frac{3}{10}, \quad P(\text{Mode}) = \frac{1}{5}$$

Let S_{at} denote the probability that a randomly selected student was satisfied with the text he/she sat.

We want $P(\text{Mean} | S_{at})$

By the version of Bayes' Theorem in the text (Formula 2-6 with $A_1 = \text{Mean}$, $A_2 = \text{Median}$, $A_3 = \text{Mode}$ so $k=3$)

$$P(\text{Mean} | S_{at}) = \frac{P(S_{at} | \text{Mean}) P(\text{Mean})}{P(S_{at} | \text{Mean}) P(\text{Mean}) + P(S_{at} | \text{Median}) P(\text{Median}) + P(S_{at} | \text{Mode}) P(\text{Mode})}$$

$$= \frac{\left(\frac{2}{5}\right) \left(\frac{1}{2}\right)}{\left(\frac{2}{5}\right) \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{3}{10}\right) + \left(\frac{4}{5}\right) \left(\frac{1}{5}\right)} = \frac{20}{51}$$

An easier way

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Use the formula (BT 2) from class

$$P(\text{Mean} | \text{Set}) = \frac{P(\text{Mean}) \cdot P(\text{Set} | \text{Mean})}{P(\text{Set})}$$

old numerator

Note $\#(\text{Set}) = 200 + 150 + 160 = 510$

so $P(\text{Set}) = \frac{510}{1000}$

Hence

$$P(\text{Mean} | \text{Set}) = \frac{\binom{1}{2}}{\binom{510}{1000}} \cdot \binom{2}{5} = \frac{\binom{2}{10}}{\binom{510}{1000}}$$

$$= \frac{\frac{200}{1000}}{\frac{510}{1000}} = \frac{20}{51}$$

5. Let $X = \#$ of girls in the family

so $P(X=2) = \frac{1}{4}$, $P(X=1) = \frac{1}{2}$, $P(X=0) = \frac{1}{4}$

We want

$$P(X=2 | X \geq 1) = \frac{P(\underbrace{(X=2)}_{(X=2) \cap (X \geq 1)})}{P(X \geq 1)}$$

($A \cap B = A$ if $A \subset B$)

$$= \frac{P(X=2)}{P(X \geq 1)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$