STAT 401 MIDTERM 1 Oct. 20, 2005

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1. The manufacturer of a certain type of automobile claims that under typical urban driving conditions the automobile will travel an average of 20 miles per gallon of gasoline. The owner of this type of automobile suspects that the mileage she is getting is not as good as the manufacturer's claim. She notes the mileages that she has obtained in her own urban driving when she fills up the tank with gasoline on nine different occasions. She finds that the results , in miles per gallon, are as follows: 15.6, 18.6, 18.3, 20.1, 21.5, 18.4, 19.1, 20.4 and 19.0.

(a) Test the manufacturer's claim versus the owner's suspicion by performing a t-test at significance level $\alpha = 0.05$. Take the manufacturer's claim as the null hypothesis and the owner's suspicion as the alternative hypothesis and assume normality of the underlying distribution of the number of miles per gallon she will get on a random trip.
State your H_0 and H_a, and the values of $\overline{x}, s, p$ and t (the test statistic). Your alternative hypothesis H_a should be *one-sided*.

(b) Compute a 95% lower-tailed confidence interval for the true average mileage per gallon.

(Warning: in part (b) you are computing a *one-sided* confidence interval. The calculator doesn't do this. So use our results from (a) to find $\overline{x}$ and s and plug them into the formula for the lower-tailed confidence interval.)

(20 points)

2. Suppose $X_1, X_2, ..., X_n$ is a random sample from a normal distribution with variance σ^2. We will assume the theorem that the ratio $Y = (\frac{n-1}{\sigma^2})S^2$ has chi-squared distribution with $n-1$ degrees of freedom.

Suppose we want to test

$$H_0 : \sigma^2 = \sigma_0^2$$

versus

$$H_a : \sigma^2 > \sigma_0^2$$

at level α.

We will use the following decision rule: reject H_0 if

$$(n-1)\frac{s^2}{\sigma_0^2} \geq \chi^2_{\alpha,n-1}$$

Let $\Psi(x)$ be the cumulative distribution function for the chi-squared distribution with $n-1$ degrees of freedom.

(a) Show that the probability of making a Type I error with this decision rule is α.

(b) Compute the $P-value$ for the above decision rule in terms of Ψ and the ratio $(n-1)\frac{s^2}{\sigma_0^2}$.

(Hint: apply Ψ to each side of the inequality $(n-1)\frac{s^2}{\sigma_0^2} \geq \chi^2_{\alpha,n-1}$ in the decision rule to find the required formula for the $P-value$ in terms of Ψ and the ratio on the left-hand side of the decision rule.)

(20 points)

3. Suppose $X_1, X_2, ..., X_n$ is a random sample from an exponential distribution with parameter λ. It can be proved that $Y = 2n\lambda\overline{X}$ has chi-squared distribution with $2n$ degrees of freedom. Recall that $1/\overline{X}$ is the point estimator for λ.

Give a decision rule for testing

$$H_0 : \lambda = \lambda_0$$

$$versus$$

$$H_a : \lambda < \lambda_0$$

at level α and prove that your test has significance level α.

(Hint: your decision rule should look like

reject H_0 if $1/\overline{x} \leq c\lambda_0$ for some constant c that you have to determine.)

(10 points)