STAT 401 **MIDTERM 1** **March 16, 2006**
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1. A contractor makes large purchases of cement from a local manufacturer. The bags of cement are supposed to weight 94 pounds. The contractor decides to test a sample of bags to see if he is getting his money's worth. He weighs 10 bags and gets the following weights: $94.1, 93.4, 92.8, 93.4, 95.4, 93.5, 94.0, 93.8, 92.9$ and 94.2.

(i) Perform the appropriate lower-tailed hypothesis test (this means test $\mu = 94$ against $\mu < 94$).
State the values of $\bar{x}, s, p$ and t. Compare p to 0.1 and decide what the contractor should conclude at the level level 0.1.

(ii) Compute a lower-tailed 90 % confidence interval for the true mean weight of a bag of cement produced by the manufacturer based on these date.

(Warning: In part (ii) you are finding a *one-sided* confidence interval for μ. The TI-83 calculator doesn't do this. So use your results from part (i) for $\bar{x}$ and s and plug them into the formula for the lower-tailed confidence interval.)

(iii) Is 94 in the lower-tailed confidence interval for μ? On the basis of your answer to this question do you accept or reject H_0 using the confidence interval test which I explained in class?

(20 points)

2. Look up the test for a population proportion on page 335 of Edition 5 or page 340 of Edition 6 and do the following problem.

(i) A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them a specified number of times and determines that 14 of the plates failed. Does this provide compelling evidence for concluding that more than 10 % of all plates fail under such circumstances. State and test the appropriate hypothesis using a significance level of .05.

Now look up the formula for β for this test on page 336 of Edition 5 or page 341 of Edition 6 and do the following problem (turn the page).

(ii) If it is really the case that 15 % of the plates will fail under such circumstances and a sample size of 100 is used, how likely is it that the null hypothesis of part (a) will not be rejected by the level .05 test.

(20 points)

3. Suppose $X_1, X_2, ..., X_n$ is a random sample from a Poisson distribution with parameter λ. It can be proved that if n is large then $Z = \frac{\overline{X}-\lambda}{\sqrt{\lambda/n}}$ has an approximately normal distribution with mean 0 and variance 1.

Give a decision rule for testing

$$H_0 : \lambda = \lambda_0$$

$$versus$$

$$H_a : \lambda < \lambda_0$$

at level α and prove that your test has approximate significance level α.

(Hint: your test should look like:
reject H_0 if $\bar{x} \leq \lambda_0 - c$ for the appropriate c depending on α, λ_0 and n.)

(10 points)