HW3, due Friday September 30
Math 403, Fall 2011
Patrick Brosnan, Instructor

Reading Assignment

Please read Chapter 2 of Herstein's book through section 2.7.

Writing Assignement

1. (25 points) A *binary operation* on a set M is simply a map $* : M \times M \to M$. Usually we write binary operations as $(x, y) \mapsto x * y$ instead of as $(x, y) \mapsto *(x, y)$. An ordered pair $(M, *)$ consisting of a set M with binary operation $*$ is sometimes called a *magma.* Sometimes, when $*$ is assumed, we just say that M is a magma.

If M is a finite set with n elements, how many binary operations are there on M?

2. (25 points) Suppose $*$ is a binary operation on a set M. An element e of M is an *identity element* for $*$ if, for all $m \in M$, $m * e = e * m = m$. Show that, if e and e' are both identity elements, then $e = e'$. In other words, show that identity elements are unique.

3. (25 points) A binary operation $* : M \times M \to M$ is *associative* if, for all $a, b, c \in M$, $(a * b) * c = a * (b * c)$. A binary operation $* : M \times M \to M$ is said to be *monoidal* if it is associative and has an identity element. In this case, the magma $(M, *)$ is said to be a monoid. How many monoidal binary operations are there on the set $M = \{1, 2\}$?

4. (25 points) Suppose G is a group with exactly two subgroups. Show that G is cyclic of prime order.

5. (10 point bonus) Herstein page 35, problem 12.