HW7, due Friday, November 4
Math 403, Fall 2011
Patrick Brosnan, Instructor

Reading Assignment

Finish reading Chapter 2.

Writing Assignement

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Problem 1. (80 points) Suppose G is a group. A subgroup H is called a *characteristic subgroup* if, for every $\phi \in \operatorname{Aut} G$, $\phi(H) = H$.

(a) Show that every characteristic subgroup is normal.
(b) Let $[G, G]$ denote the commutator subgroup of G. That is, $[G, G]$ is the subgroup of G generated by elements of the form $ghg^{-1}h^{-1}$ for $g, h \in G$. Show that $[G, G]$ is characteristic.
(c) Suppose G is an abelian group and n is a positive integer. Set $G[n] := \{g \in G : g^n = e\}$. Show that $G[n]$ is characteristic.
(d) List all of the subgroups of $\mathbf{Z}/2 \times \mathbf{Z}/2$. Which are characteristic? Which are not?

Problem 2. (20 points) Suppose G is the dihedral group of order $2n$ (from HW 6). What is the commutator subgroup of G?

Bonus. (20 bonus points) Do the same as in 1 (d) for $G = \mathbf{Z}/6 \times \mathbf{Z}/3$. Then do the same again for $G = \mathbf{Z}/4 \times \mathbf{Z}/2$.