HW3, due Tuesday, October 8
Math 403, Fall 2013
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1. Compute the subgroups $\langle R_{90}, H\rangle$ and $\langle H, D\rangle$ of the dihedral group of order 8.

2. What is the center of the dihedral group of order 8?

3. On scratch paper, which you do not have to turn in, make a table of all the elements of the symmetric group S_4 with their orders. (This may take a while, but it should be good practice.) Then turn in a table which lists the number of elements in S_4 of each order.

4. Suppose G is a group and S is a subset of G. We say that G is *generated* by S if $G = \langle S\rangle$. Show that the group S_3 can be generated by 2 elements of order 2.

5. List all cyclic subgroups of $U(30)$.

6. Suppose G is a group and g is an element of g. The *centralizer* of g is the set $C(g)$ of all elements $h \in G$ such that $gh = hg$. Show that the centralizer $C(g)$ is a subgroup of G.

7. Write $\mathbf{GL}_2(\mathbb{R})$ for the group of all invertible 2×2-invertible matrices with real entries. Let

$$X := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
$$Y := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Compute $C(X)$ and $C(Y)$. Is $C(X)$ abelian? What about $C(Y)$?

8. Suppose G is a group with elements x, y and z.
(a) Show that $(xyx^{-1})(xzx^{-1}) = xyzx^{-1}$.
(b) Show that $xyx^{-1} = e \Leftrightarrow y = e$.
(c) Show that $|y| = |xyx^{-1}|$.

9. Suppose G is a group with identity element e and $g \in G$. Show that $g = g^{-1}$ if and only if $g^2 = e$. Using this observation, show that any finite group of even order contains an element of order 2.

10. Suppose G is a cyclic group. We say that an element g of G is a *generator* of G if $G = \langle g\rangle$.
(a) Find all generators of the group $\mathbb{Z}$ of integers?
(b) Find all generators of the group $\mathbb{Z}/7$.