HW4, due Thursday, October 17
Math 403, Fall 2013
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1 (10 points). Let $\sigma \in S_5$ denote the element

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}.$$

Write σ as a product of disjoint cycles.

2 (10 points). What is the order of the element σ in Problem 1?

3 (20 points). What is the maximal possible order an element in S_7?

4 (20 points). Suppose that $\alpha, \beta \in S_n$. If α is even, show that $\beta\alpha\beta^{-1}$ is also even.

5 (20 points). Suppose n is an integer greater than 2. Write Z_n for the group $\mathbb{Z}/n$, and write $G_n = U(n) \times Z_n$. Here $U(n)$ denotes the group of $[a] \in \mathbb{Z}/n$ such that $(a,n) = 1$ under multiplication. Define a binary operation on G_n by setting

$$(a,x)(b,y) = (ab, ay+x)$$

for $a, b \in U(n)$ and $x, y \in Z_n$. Show that, with the above binary operation, G_n is a group.

6 (20 points). Let $H_n = \{(a,b) \in G_n : a = \pm 1\}$. Show that H_n is a subgroup of G_n. Then show that H_n is isomorphic to the dihedral group of order $2n$.