HW5, due Tuesday, November 5
Math 403, Fall 2013
Patrick Brosnan, Instructor

Practice Problems: Do the following problems from Gallian for practice, but do not turn them in. The format below is that **G4** means "Chapter 4 of Gallian."

G4: 1, 3, 5, 13, 45
G5: 1, 3, 5, 11, 31
G6: 1, 5
G7: 11, 17, 31

1. Suppose G is a group and H and K are two subgroups. Suppose further that K has prime order and that H does not not contain K. Show that $H \cap K = \{e\}$.

2. Show that any group of order 275 has an element of order 5.

3. Show that the mapping of $U(17)$ to itself given by $x \mapsto x^3$ is an automorphism.

4. Let G_n be the group from Problem 4 of HW4, and suppose n is odd. Let H_n be the group from Problem 5 of HW4, which is isomorphic to the dihedral group with $2n$ elements.

    (a) Show that, if $g \in G_n$ and $h \in H_n$, then $ghg^{-1} \in H_n$.

    (b) Use part (a) to show that, for $g \in G_n$, the map $\varphi_g : H_n \to H_n$ given by $h \mapsto ghg^{-1}$ is a group automorphism.

    (c) (10 bonus points) Show that every group automorphism of H_n is of the form φ_g for some $g \in G_n$. And conclude that $\operatorname{Aut} D_n \cong G_n$.