

HW6, due Tuesday, November 19

Math 403, Fall 2013

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Practice Problems: Do the following problems from Gallian for practice, but do not turn them in. The format below is that **G4** means “Chapter 4 of Gallian.”

G8: 1, 3, 8, 9, 31, 39, 53

G9: 1, 7, 11, 31

G10: 5, 9, 17, 31

1. Let $U(9)$ act on Z_9 by multiplication. What are the orbits? What are the stabilizers?
2. Suppose p is an odd prime. Show that $U(p)$ is isomorphic to $U(2p)$.
3. Show that $Z_4 \times Z_6 \cong Z_{12} \times Z_2$.
4. Suppose G is an abelian group and n is a positive integer. Set $G[n] = \{g \in G : g^n = e\}$. Show that $G[n]$ is a subgroup of G . Then use the to show that $G_{\text{tors}} := \cup_{n>1} G[n]$ is a subgroup of G . This group is called the *torsion subgroup* of G .
5. Let G denote the group \mathbb{C}^\times of all non-zero elements of \mathbb{C} with multiplication as the binary operation. Show that, for each n , $G[n]$ is cyclic of order n . It might be helpful to use the facts about complex numbers in the Chapter 0 of the text.