HW3, due Wednesday, February 19
Math 403, Spring 2014
Patrick Brosnan, Instructor

1. Suppose G is a group and $g \in G$. The *centralizer* of $g \in G$ is the subset $C(g) = \{h \in G : gh = hg\}$. If S is a subset of G, then the centralizer of S is the subset $C(S) = \cap_{g \in S} C(g)$.

(1) Show that $C(g) \leq G$ for every $g \in G$.
(2) Show that $C(S) \leq G$ for every subset S of G.
(3) Suppose $G = \mathbf{GL}_2(\mathbb{R})$. What is the centralizer of the matrix
$$g = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}?$$
(4) In $\mathbf{GL}_2(\mathbb{R})$, what is the centralizer of
$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}?$$
(5) Again in $\mathbf{GL}_2(\mathbf{R})$, what is the centralizer of
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}?$$

2. Suppose G is a group. The center of G is the subset $Z(G) = \cap_{g \in G} C(g)$. Show that the center is a subgroup of G. Is the center abelian?

3. Suppose G is a group and $g \in G$. Is $C(g)$ always abelian?

4. Suppose G is a group and S is a subset of G. We say that S is a set of commuting elements of G if, for all $x, y \in S$, $xy = yx$. Show that if S is a set of commuting elements then the subgroup $\langle S \rangle$ is abelian.