HW6, due Wednesday, April 2
Math 403, Spring 2014
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1. Suppose G is a an abelian group of order 8. Suppose $g^2 = e$ for every element of G. Show that $G \cong C_2 \times C_2 \times C_2$.

2. Suppose G is a non-cyclic abelian group of order 8, and suppose that G contains an element x of order 4. Set $H = \langle x \rangle$.

(1) Show that $g^2 \in H$ for all $g \in G$. (**Hint:** H is normal and G/H has order 2.)
(2) Show that, if g is as above, then g^2 is either e or x^2.
(3) Show that there exists a $y \in G$ such that $y \notin H$ and $y^2 = e$.
(4) Set $K = \langle y \rangle$ with y as above. Show that G is the internal direct product of H and K, and conclude that $G \cong C_4 \times C_2$.

3. Suppose that G is an abelian group or order 8. Show that G is isomorphic to exactly one of the following: $C_8, C_4 \times C_2, C_2 \times C_2 \times C_2$.

4. Suppose G is a non-abelian group of order 8 and suppose G has at least 2 elements of order 2.

(1) Show that G has an element x of order 4 and an element y of order 2 such that $y \notin \langle x \rangle$.
(2) Show that, with x and y as above, $yxy = x^{-1}$.
(3) (**Bonus:** 5 points) Show that $G \cong D_4$.