**HW2, due Wednesday, March 1**
**Math 403, Spring 2017**
**Patrick Brosnan, Instructor**

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**Practice Problems:** Do the following problems from Herstein for practice, but do not turn them in. The format below is that **H4.5** means "Chapter 4, Section 5 of Herstein."

**H2.2:** 1, 2, 3
**H2.3:** 14, 16, 19
**H3.1:** 1, 2
**H3.2:** 2, 3, 14

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**Graded Problems:** Work the following problems for a grade.

**1.** Suppose $n$ is a non-zero integer and $k$ is an integer. Show that $n/(n,k)$ is relatively prime to $k/(n,k)$.

**2.** Suppose $G = \langle g \rangle$ is a cyclic group of order $n$ with $e$ as the identity element. Let $k$ be an integer, and set $h = g^k$. Show that

$$h^i = e \Leftrightarrow \frac{n}{(n,k)} | i.$$

Conclude that $|h| = n/(n,k)$.

**3.** Recall the group $\mathbf{O}_2(\mathbb{R})$ from Problem set 2. Show that, for any $\theta \in \mathbb{R}$, $TR(\theta)$ is its own inverse.

**4.** Let $n$ be a positive integer and let $R = R(2\pi/n)$. Let $D_n := \langle T, R \rangle$ denote the subgroup of $\mathbf{O}_2(\mathbb{R})$ generated by $R$ and $T$. The group $D_n$ is called the *dihedral group*.

(1) Show that $|D_n| = 2n$.
(2) Show that $D_n$ is abelian if and only if $n < 3$.
(3) Set $\mathbf{e} = (1,0)$ and let $P_n = \{R^k\mathbf{e} : k \in \mathbb{Z}\}$. Show that $P_n$ consists of $n$ points lying on the circle.
(4) Show that $TR^k e = R^{-k}e$.
(5) **10 point Bonus:** Show that $D_n$ is the set of all $X \in \mathbf{O}_2(\mathbb{R})$ such that $X(P_n) = P_n$.

**5.** What is the maximal order of an element of $S_5$?