HW5, due Friday, March 17
Math 403, Spring 2017
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Practice Problems: Do the following problems from Herstein for practice, but do not turn them in. The format below is that **H4.5** means "Chapter 4, Section 5 of Herstein."

H2.7: 5
H2.9: 1, 3

Graded Problems: Work the following problems for a grade.

1. Show that the dihedral group D_4 is isomorphic to the subgroup P of S_4 generated by (1234) and (13). **(Hint:** Use the result of Problem 4.5 in HW 3.) Moreover, show that there is an isomorphism $\varphi : D_4 \to P$ such that $\varphi(R) = (1234)$ and $\varphi(T) = (13)$.

2. List the left and right cosets of the subgroup P of D_4 in Problem 1. Is P normal?

3. Let U denote the set of 2×2 matrices of the form

$$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$$

with $x \in \mathbb{R}$.

(1) Show that U is a subgroup of $\mathbf{GL}_2(\mathbb{R})$.
(2) Show that U is not normal.
(3) Show that U is isomorphic to $\mathbb{R}$ as a group.

4. Suppose G is a group, $H \leq G$ and $x \in G$. Show that $xHx^{-1} \leq G$.

5. Suppose G is a group and H, K are subgroups of G. We say that H is *conjugate* to K if $H = xKx^{-1}$ for some $x \in G$.

(1) Show that the property of being conjugate is an equivalence relation on the set of subgroups of G.
(2) Show that any two conjugate subgroups are isomorphic as groups.
(3) Show that the subgroups U of Problems 3 is conjugate to the subgroup L consisting of all matrices of the form

$$\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}.$$