HW7, due Friday, April 7
Math 403, Spring 2017
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Graded Problems: Work the following problems for a grade.

1. Suppose G is a group in which every element has order dividing 2. Show that G is abelian.

2. Suppose that G is a group and H is a subgroup of index 2. Show that H is normal.

3. Suppose H and K are groups. Write $\mathrm{Hom}(H,K)$ for the set of homomorphisms from H to K.

(a) Suppose n is a positive integer, and $f \in \mathrm{Hom}(\mathbb{Z}/n\mathbb{Z}, K)$. Show that $f([1])^n = e$.

(b) Write $K[n] = \{k \in K : k^n = e\}$. Then part (1) gives a map, $\mathrm{Hom}(\mathbb{Z}/n\mathbb{Z}, K) \to K[n]$ given by $f \mapsto f([1])$. Show that this map is an isomorphism of sets. In other words, show that, for every $k \in K[n]$, there exists a unique group homomorphism $f : \mathbb{Z}/n\mathbb{Z} \to K$ such that $f([1]) = k$.

4. Suppose n is a positive integer. Show that S_{2n} contains a subgroup H which is isomorphic to C_2^n.

5. Suppose that G is a group and $N_1, \ldots, N_r$ are normal subgroups. Suppose that $G = N_1 \cdots N_r$ and, for each $i = 2, \ldots, r$, $(N_1 \cdots N_{i-1}) \cap N_i = \{e\}$. Show that G is the (internal) direct product of $N_1, \ldots, N_r$.