HW1, due Wednesday, February 18
Math 404, Spring 2014
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1. Identify $\mathbb{R}^2$ with the field $\mathbb{C}$ of complex numbers via the map $(x,y) \mapsto x+iy$. Let S be a subset of $\mathbb{C}$ containing 0 and 1, and let $C(S)$ denote the set of elements of $\mathbb{R}^2$ constructible from S by straight-edge and compass. Show that $C(S)$, regarded as a subset of $\mathbb{C}$, is actually a subfield of $\mathbb{C}$.

2. Again suppose that S is a subset of $\mathbb{C}$ containing 0 and 1. Suppose $z \in C(S)$. Show that $\pm\sqrt{z}$ are in $C(S)$ as well.

3. Set $E = \mathbb{Q}(\sqrt{3}, \sqrt{5}) \subset \mathbb{R}$. Compute $[E : \mathbb{Q}]$.

4. Suppose F is a field and $f \in F[x]$ is an irreducible polynomial of degree n. Suppose E/F is a finite field extension with $[E : F] = m$ and $\gcd(m,n) = 1$. Show that f is also irreducible when regarded as a polynomial in $E[x]$.