HW9, due Monday, May 11
Math 404, Spring 2015
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1. Use the formulas for the discriminant (for example, on page 48 of Milne's text) to compute the Galois groups of the following polynomials over $\mathbb{Q}$.

    (a) $f(x) = x^3 + 7x + 2$.
    (b) $g(x) = 27x^3 - 63x - 7$.

2. Suppose G is a solvable group and $G \to Q$ is a surjective group homomorphism. Show that Q is solvable.

3. Let S_n denote the symmetric group on $n \geq 2$ letters. Using the fact that S_n is generated by transpositions (ij) for $1 \leq i < j \leq n$, prove the following.

    (1) S_n is generated by transpositions of the form $(1i)$ for $1 < i \leq n$.
    (2) S_n is generated by transpositions of the form $(i, i+1)$ for $1 \leq i < n$.

4. Show that the equation $2x^5 - 10x + 5$ is irreducible over $\mathbb{Q}$ and has exactly three real roots.