HW2, due Wednesday, September 16
Math 404, Spring 2014
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Exercises from the Hoffman-Kunze: (16 points each) 1.6.1, 1.6.5, 1.6.6, 1.6.7

1 (20 points)**.** Let $\mathbb{F}_2$ denote the field with two elements, and let X be a set. Write $P(X)$ for the set of subsets of X. Suppose $S,T \in P(X)$. Define $S+T$ to be the set of all $x \in X$ such that x is in either S or in T but not in both. So, symbolically,

$$S+T := S \cup T \setminus S \cap T.$$

Define a map $\mathbb{F}_2 \times P(X) \to P(X)$ by setting $1S = S$ and $0S = \emptyset$. Show that, with the operations of addition and multiplication by $\mathbb{F}_2$ just given, $P(X)$ is a vector space over $\mathbb{F}_2$.