Math 241: Matlab Project 2
due in the discussion session May 13

You first have to download the files `plotpts.m`, `fillpts.m`, `nice3d.m` , `parallepip.m` from the course web page. Use the command `nice3d` after the plotting commands.

Remember that you can work in teams of up to 3 students. Sharing of material between different teams is not permitted.

1. We want to throw a ball as far as possible. We throw the ball from a height of 8 feet with an initial speed of $V_0 = 16$ feet per second. At what angle θ should we throw the ball? The x-axis is horizontal, the y-axis is vertical.

- **(a)** According to Newton's law we have $\mathbf{r}''(t) = (0, -32)$. We have $\mathbf{r}(0) = (0, 8)$. Assume the initial velocity is $\mathbf{r}'(0) = (a, b)$ and find $\mathbf{r}(t) = (x(t), y(t))$ as an expression of a, b, t.
- **(b)** Find the time $T > 0$ when the ball hits the ground, i.e., $y(T) = 0$. Then find the distance $x(T)$ as an expression of a, b. We call this expression $f(a, b)$ (which we later want to maximize).
- **(c)** We throw the ball at an angle θ so that $a = V_0 \cos\theta$, $b = V_0 \sin\theta$ where $V_0 = 16$. Try out the angles 10°, 20°,...,80°: For $\theta = \frac{\pi}{2} \cdot \frac{j}{9}$ and $j = 1, \ldots, 8$ find the distance $f(a, b)$ and plot the curve $\mathbf{r}(t)$ for $t \in [0, T]$ (plot these 8 curves together in the same graph). Which of these angles gives the largest distance?
- **(d)** We want to find a, b such that $f(a, b)$ is maximal, subject to the constraint $a^2 + b^2 = 16^2$. Use Lagrange multipliers to find the optimal a, b. What is $\theta = \arctan(b/a)$?

2. For the following problem use the symbolic integration command `int` and give the results $V, \bar{x}, \bar{y}, \bar{z}$ as symbolic expressions. Then use `double()` to find numerical values.

- **(a)** Consider the cylinder consisting of points $(x, y, z) \in \mathbb{R}^3$ satisfying $x^2 + z^2 \leq 1$. Let D denote the part of this cylinder with $-x \leq y \leq x$, $z \geq 0$.
  Plot the top surface of the region D using `ezsurfvs`.
  Find the volume V of D and the center of mass $(\bar{x}, \bar{y}, \bar{z})$ (assuming density 1).
- **(b)** In cylindrical coordinates (r, θ, z) a torus is described by $(r - 2)^2 + z^2 \leq 1$. Let D denote the part of this torus with $x \geq 0$, $y \geq 0$, $z \geq 0$.
  Plot the top surface of the region using `ezsurfpol`.
  Find the volume V of D and the center of mass $(\bar{x}, \bar{y}, \bar{z})$ (assuming density 1).