Math 241: Practice Problems for Exam 1

1. Consider the points $\mathbf{P} = (1, 1, 2)$, $\mathbf{Q} = (2, 1, 1)$, $\mathbf{R} = (1, 2, 1)$

- **(a)** Find the symmetric equations of the line through the points $\mathbf{P}$ and $\mathbf{Q}$.
- **(b)** For the point $\mathbf{R}$ find the closest point $\mathbf{S}$ on the line from (a).
- **(c)** Find the area of the triangle with the corners $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$.
- **(d)** Find the volume of the tetrahedron with the vertices $(0, 0, 0)$, $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$.
- **(e)** Find an equation $Ax + By + Cy = D$ for the plane through the points $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$.
- **(f)** Find an equation $A'x+B'y+C'y = D'$ of the plane through the points P, Q which is orthogonal on the plane from (e).

2. Consider the position function $\mathbf{r}(t) = (\frac{1}{3}t^3,\, 2t,\, t^2)$.

- **(a)** For the time $t = -1$ find the speed vector $\mathbf{v}_0 = \mathbf{v}(-1)$ and the acceleration vector $\mathbf{a}_0 = \mathbf{a}(-1)$.
- **(b)** Find the decomposition $\mathbf{a}_0 = \mathbf{a}_{\text{par}} + \mathbf{a}_{\text{orth}}$ where $\mathbf{a}_{\text{par}}$ is parallel to $\mathbf{v}_0$ and $\mathbf{a}_{\text{orth}}$ is orthogonal on $\mathbf{v}_0$. Use this to find the change of speed $V'(-1)$ and the curvature $\kappa(-1)$ (here $V(t)$ denotes the speed).
- **(c)** Consider the curve given by $\mathbf{r}(t)$ for t between -1 and 1 and find the length of this curve.

3. Assume that the acceleration vector is $\mathbf{a}(t) = (1, t, -1)$. The initial position is $\mathbf{r}(0) = (1, 0, 0)$, the initial velocity is $\mathbf{v}(0) = (1, 0, 1)$. Find the position $\mathbf{r}(t)$.