Practice Problems for Exam 4

1. Let $\mathscr{C}$ denote the helix given by $(2\cos t, 2\sin t, t)$ for $t \in [0, 2\pi]$

(a) Find $I = \displaystyle\int_{\mathscr{C}} (x+y+z)ds$.

(b) Find $W = \int_{\mathscr{C}} x\,dx + y\,dy + z\,dz$.

(c) Show that the vector field from (b) is conservative. Find the integral W using the fundamental theorem of line integrals.

2. Let $\mathscr{C}$ denote the closed curve given by the points (x, y) with $x^2 + y^2 = 4$, traversed counterclockwise.

(a) Find $W = \int_{\mathscr{C}} (x-y)dx + (x+y)dy$.

(b) Find the integral W from (a) using Green's theorem.

3. Let Σ be the part of the sphere $x^2+y^2+z^2 = 4$ with $x \geq 0$, $y \geq 0$, $z \geq 0$. Write the surface integral $\displaystyle\iint (x+y+z)dS$ as an iterated integral over θ, ϕ and evaluate it.

4. Let Σ denote the whole sphere $x^2+y^2+z^2 = 4$ and consider the vector field $\vec{F} = (x, y, z)$.

(a) Write the flux integral $I = \iint_{\Sigma} \vec{F} \cdot \vec{n}\,dS$ as an iterated integral over ϕ, θ and evaluate it.

(b) Evaluate the integral I from (a) using the divergence theorem.