Topics for Exam 4

Parametrizations you should know:

straight line from $\vec{A}$ to $\vec{B}$: $\vec{r}(t) = \vec{A} + (\vec{B} - \vec{A})t$ with $0 \le t \le 1$

circle $x^2 + y^2 = R^2$, $z = 0$: $\vec{r}(t) = (R\cos t, R\sin t, 0)$ with $0 \le t \le 2\pi$

sphere $x^2 + y^2 + z^2 = R^2$: $\vec{r}(\phi, \theta) = (R\sin\phi\cos\theta, R\sin\phi\sin\theta, R\cos\phi)$ with $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$

cylinder $x^2 + y^2 = R^2$, $a \le z \le b$: $\vec{r}(\theta, z) = (R\cos\theta, R\sin\theta, z)$ with $0 \le \theta \le 2\pi$, $a \le z \le b$

graph of function: points $z = f(x,y)$ for $a \le x \le b$, $c \le y \le d$: $\vec{r}(x,y) = (x, y, f(x,y))$ with $a \le x \le b$, $c \le y \le d$

Line integrals

scalar function f: $\boxed{ds = \|\vec{r}'\|\,dt}$

"work integral" for vector field $\vec{F}$: $\boxed{d\vec{r} = \vec{r}'\,dt}$

Surface integrals

scalar function f: $\boxed{dS = \|\vec{r}_u \times \vec{r}_v\|\,du\,dv}$

"flux integral" for vector field $\vec{F}$: $\boxed{\vec{n}\,dS = (\vec{r}_u \times \vec{r}_v)du\,dv}$

Finding potential f for vector field $\vec{F}$: If $\operatorname{curl}\vec{F} = \vec{0}$ there exists a potential $f(x,y,z)$ (in domain without "holes")

1. $f_x = F_1$: taking antiderivative w.r.t. x gives $f(x,y,z) = (\cdots) + g(y,z)$
2. $f_y = F_2$ gives $g_y(y,z) = \cdots$, taking antiderivative w.r.t. y gives $g(y,z) = (\cdots) + h(z)$
3. $f_z = F_3$ gives $h'(z) = \cdots$, taking antiderivative w.r.t. z gives $h(z) = (\cdots)$ (we can use constant $C = 0$)

Fundamental theorem of line integrals: $\boxed{\int_{\mathscr{C}} \operatorname{grad} f \cdot d\vec{r} = f(\vec{B}) - f(\vec{A})}$

The curve $\mathscr{C}$ starts at point $\vec{A}$ and ends at point $\vec{B}$.

Green's theorem: $\boxed{\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA = \int_{\mathscr{C}} (F_1 dx + F_2 dy)}$

The curve $\mathscr{C}$ is the boundary of the 2D region R, oriented counter clockwise.

Divergence theorem: $\boxed{\iiint_D \operatorname{div}\vec{F}\,dV = \iint_\Sigma \vec{F} \cdot \vec{n}\,dS}$

The surface Σ is the boundary of the 3D region D, with the normal vector $\vec{n}$ pointing outside.