Assignment #1 MATH246, Spring 2024
due 02/16 at 10pm

1. Do these problems with paper and pencil **without Matlab**.
   Find the **solution** of the initial value problem and its **interval of definition**. Explain what the type of the ODE is and show all the steps for finding the answers.

   (a) $y' = \frac{y}{t} + 1$, $y(1) = 2$

   (b) $y' = \frac{y^2}{t}$, initial condition (i) $y(1) = 2$, (ii) $y(1) = 0$.

   (c) $y' = -\frac{2t+y}{t+2y}$, $y(1) = 2$. Hint: Try to write this as $M(t,y)dt + N(t,y)dy = 0$. First find the solution in implicit form, then solve for $y(t)$.

   (d) $t^2y + 2ty^2 + (t^2y + y^2)y' = 0$, $y(0) = 1$.

2. Let $g(y) = y(y-1)(y-3)$ and consider the ODE $y'(t) = g(y(t))$. Do **NOT** find a formula for the solution of this ODE.

   (a) Sketch the graph of $g(y)$ and the direction field for the ODE **by hand**. What are the stationary solutions?

   (b) We consider the initial conditions (i) $y(0) = -1$, (ii) $y(0) = \frac{1}{2}$, (iii) $y(0) = 2$, (iv) $y(0) = 4$. Explain in each case what happens with the solution $y(t)$ as $t \to \infty$ and as $t \to -\infty$.

3. We want to find a function $y(x)$ such that $2xy^3 + 1 + (3x^2y^2 + 2y)y' = 0$.

   (a) (**Without Matlab**): Find the solution $H(x,y) = c$ in implicit form.

   (b) Use `fcontour` to plot the contours $H(x,y) = c$ for $x \in [-2, 2]$, $y \in [-2.5, 1.5]$ (**use these limits for all graphs for problem 3**). Use `'LevelStep',...` to show more contours.

   (c) Now consider the initial condition (i) $y(0) = 1$ and (ii) $y(0) = -1$. Find the value of c for each initial condition. Use `fcontour(...,'LevelList',c)` to plot the curves $H(x,y) = c$ with this value of c, mark the two initial points (x_0, y_0) with circles. Look at this graph: what interval of definition would you expect for initial condition (i) and for initial condition (ii)?

   (d) Write the ODE in the form $y' = f(x,y)$. Use **`dirfield`** to plot the direction field.
   Use **`ode15s`** to find the solution for $x \in [-2, 2]$, or as far as it exists to the left and right. Plot this solution for initial condition (i) in blue, mark the initial point (x_0, y_0) with a circle.
   Repeat this for initial condition (ii), but plot everything in green. You should make a single plot for (d) containing the direction field, and the two solutions for (i) and (ii).