Exam 2

You must **show all your work** in order to get credit.
READ THE WHOLE PROBLEM BEFORE YOU START WORKING!

1. **(10 pts)** Find the correct form of the particular solution $Y(t)$ using the **method of undetermined coefficients**, but **DO NOT determine the coefficients** A_j, B_j etc.:

$$y'' + 2y' + y = -t - t \cdot e^{-t} - t \cdot e^{-t} \cdot \sin t$$

2. **(20 pts)** Find the particular solution $Y(t)$ using the method of **variation of parameters** and **simplify the answer**. (**NO credit** if you use another method).

$$y'' - 4y = 1 + e^{2t}$$

3. **(20 pts)** Consider a mass-spring-damper system with a mass $m = 2$. The external force is $F(t) = 2\cos(3t)$.

   (a) (10 pts) Assume that there is no damping, i.e., $c = 0$. Find a spring constant k so that there is resonance. Find the general solution, but **DO NOT determine the constants A, B in the method of undetermined coefficients**. Describe the behavior of the solution as t goes to infinity.

   (b) (10 pts) Assume that the damping constant is $c = 1$. Find a spring constant k so that there is critical damping. Find the general solution, but **DO NOT determine the constants A, B in the method of undetermined coefficients**. Describe the behavior of the solution as t goes to infinity.

4. **(50 pts)**

   (a) (25 pts) Consider the IVP

$$y'' + 2y' + 3y = f(t), \qquad f(t) = \begin{cases} t & \text{for } t < 1 \\ e^{-t} & \text{for } t \geq 1 \end{cases}, \qquad y(0) = -2, \quad y'(0) = 3.$$

   Find $Y(s)$, i.e., the Laplace transform of the solution $y(t)$. **DO NOT find $y(t)$. DO NOT simplify $Y(s)$.**

   (b) (25 pts) Find the inverse Laplace transform of the function

$$Y(s) = \frac{e^{-s}}{s(s^2 + 2s + 5)}$$

function	Laplace transform
$t^k e^{at}$	$\frac{k!}{(s-a)^{k+1}}$ $\quad k = 0, 1, 2, \ldots$. Note that $0! = 1$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
$u(t-c)f(t-c)$	$e^{-cs}F(s)$