Practice problems for Exam 2

1. Find the general solution of the ODE $y'' - 2y' + y = e^t$.

2. Find the general solution of the ODE $y'' - 2y' + y = t^{1/2}e^t$.

3. We have a mass-spring-damper system with mass $m = 2$ and spring constant $k = 8$.

(a) Determine the damping constant c so that there is critical damping. Then determine the steady-state solution $Y(t)$ for the external force $F(t) = 2$.

(b) Assume that there is no damping ($c = 0$) and that the external force is $F(t) = 2\sin(\omega t)$. Determine ω so that there is resonance.

4.

(a) For the IVP

$$y'' + 3y' + 5y = f(t) = \begin{cases} t^2 & \text{for } t < 2 \\ 4 & \text{for } t \geq 2 \end{cases}, \qquad y(0) = 3, \quad y'(0) = -2$$

determine $Y(s)$ (the Laplace transform of the solution $y(t)$). DO **NOT** find $y(t)$. DO NOT simplify the result.

(b) Find the inverse Laplace transform $y(t)$ for

$$Y(s) = \frac{1+s}{s(s^2 - 2s + 2)}$$

(c) Find the inverse Laplace transform $y(t)$ for

$$Y(s) = e^{-3s}\frac{4}{s^2(s^2 - 4)}$$