Exam 3 MATH246, Spring 2024

You must **show all your work** in order to get credit.
Read the whole problem carefully before you start working.
All GRAPHS must have at least HALF THE WIDTH OF THE PAGE.

1. **(50 pts)** Consider the ODE system $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1-2y \\ x^2-y^2 \end{bmatrix}$.

   (a) (20 pts) Find all stationary points (there are two), and the Jacobian matrix A at each stationary point.

   (b) (20 pts) **For the linearized problem at each stationary point:** find the **type** (include clockwise/counterclockwise where appropriate) and **stability** (stable/unstable, attracting?, repelling?). Sketch a **phase portrait**. **Hint:** You only need to find eigenvectors for real eigenvalues.

   (c) (10 pts) What can you conclude for the stationary points of the nonlinear system? Sketch a possible **phase portrait** which indicates how the trajectories for the stationary points may connect.

2. (**25 pts**) Consider the ODE system $\vec{y}\,' = \begin{bmatrix} 6 & 2 \\ 5 & 9 \end{bmatrix} \vec{y}$. Find the **general solution** (using real valued functions) of the ODE system. Find the **type** (include clockwise/counterclockwise where appropriate) and **stability** (stable/unstable, attracting?, repelling?) of the stationary point $(0,0)$. Sketch the **phase portrait**.

3. (**25 pts**) Consider the ODE system $\vec{y}\,' = \begin{bmatrix} 0 & -2 \\ 8 & 0 \end{bmatrix} \vec{y}$. Find the **general solution** (using real valued functions) of the ODE system. Find the **type** (include clockwise/counterclockwise where appropriate) and **stability** (stable/unstable, attracting?, repelling?) of the stationary point $(0,0)$. Sketch the **phase portrait**.