Practice problems for Exam 1

1. Approximate $y = (3.5)^{1/2}$ using the Taylor polynomial $p_2(x)$. Give an upper bound $|y - p_2(x)| \le \cdots$.

2. We use the following Matlab command: `y = 1000.2 - 1000.1`
Give an upper bound for the relative error of the computed result

3. We want to compute $y = e^{.001} - 1$ and use the Matlab code `y = exp(.001) - 1`

(a) Which operation (exp or subtraction) will cause a large magnification of the relative error? Find the magnification factor (condition number) for this operation, give the approximate answer as a number like $3 \cdot 10^7$. *Hint:* Use a Taylor approximation for $e^{.001}$ to evaluate your expression for the error.

(b) Can we get a more accurate result if we evaluate the Taylor approximation $p_3(x)$ in Matlab?

4. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 4 \\ 4 & 1 & 2 \end{bmatrix}$

(a) Use Gaussian elimination WITH pivoting (use the pivot candidate with the largest absolute value) to find the matrices L, U and the vector p .

(b) Use L, U, p to solve the linear system $Ax = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$.

(c) We solve the linear system $Ax = \begin{bmatrix} 1 \\ 10 \\ 1000 \end{bmatrix}$ and find the solution vector x. Then we find out that we actually need the solution vector $\tilde{x}$ for the linear system $A\tilde{x} = \begin{bmatrix} -1 \\ 10 \\ 1000 \end{bmatrix}$. Find an upper bound $\dfrac{\|\tilde{x} - x\|_\infty}{\|x\|_\infty} \le \cdots$ assuming $\|A^{-1}\|_\infty \le 10$.