Assignment #1, due Thursday, Feb. 12

Do all problems with pencil and paper unless I ask you to use Matlab.

1. Consider the vectors $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} -1 \\ t \\ 0 \end{bmatrix}$ where (i) $t = 1$, (ii) $t = -1$. For each case (i), (ii): Are the vectors linearly dependent? If yes, find c_1, c_2, c_3 which are not all zero such that $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = \vec{0}$. If no, prove that $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = \vec{0}$ implies $c_1 = c_2 = c_3 = 0$.

2. Consider the subspace $U = \operatorname{span}\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix} \}$. Select some of these vectors such that (i) the vectors are linearly independent and (ii) they span the same subspace U.

3. Consider the matrix $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 4 & 1 & t & 1 \end{bmatrix}$ where (i) $t = 2$, (ii) $t = 1$. For each case (i), (ii):

Perform Gaussian elimination as far as possible, **choose the first nonzero element among the pivot candidates as pivot**. Is the matrix A singular or nonsingular?

- **If the matrix is singular:** use the resulting matrix from the elimination to find a vector $\vec{x} \neq \vec{0}$ such that $A\vec{x} = \vec{0}$.
- **If the matrix is nonsingular:** check that LU gives the expected result. Use L, U, p to solve the linear system $Ax = [2, 3, 2, 1]^\top$.
  Then solve the linear system in Matlab using `[L,U,p]=lu(A,'vector')` . Does Matlab get the same L, U, p as you did? Explain!

4. Consider the three linear systems

(i) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, (ii) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, (iii) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

For each linear system do the following:

(a) Find the set of all $\vec{x} \in \mathbb{R}^2$ which satisfy the linear system.

(b) Use Matlab to plot the two lines corresponding to the two equations together. See the instructions on the web page. Explain how you can see the solution set.

5. Consider the two linear systems

(i) $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, (ii) $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$

For each linear system do the following:

(a) Find the set of all $\vec{x} \in \mathbb{R}^3$ which satisfy the linear system by hand.

(b) Use Matlab to plot the three planes corresponding to the three equations together. See the instructions on the web page. Explain how you can see the solution set.