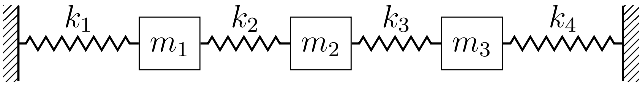
Assignment #2, due Thursday, May 7

1. We consider three masses and four springs with spring constants k_1, k_2, k_3, k_4. The masses can only move horizontally. This is the picture at equilibrium:



Let $m_1 = m_2 = m_3 = 1$ and $k_1 = 2$, $k_2 = k_3 = 1$, $k_4 = 2$.

(a) We pull with horizontal forces F_1, F_2, F_3 at the three masses. Then the masses will move to new equilibrium positions. We want to know the resulting horizontal displacements x_1, x_2, x_3 of the masses from their original positions. Write down the linear system $A\vec{x} = \vec{F}$ with a 3×3 matrix A. Use Matlab to find the answer for $\vec{F} = [4, 2, 4]^\top$.

(b) Now we consider the time dependent problem with $\vec{F} = \vec{0}$. Find the eigenmodes of the form $\cos(\omega t)\vec{v}$ by hand (hint: the eigenvalues are small integers). Check your answer in Matlab using symbolic matrices.

(c) Now consider the problem with $\vec{F} = \vec{0}$ and initial conditions

$$\vec{x}(0) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \qquad \vec{x}'(0) = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

Write down the general solution with parameters c_1, c_2, c_3 and d_1, d_2, d_3. Write down the linear system for $\vec{c}$ and for $\vec{d}$. Then use Matlab to find $\vec{c}, \vec{d}$.

(d) Now consider the problem with $\vec{F} = [4, 2, 4]$ and the same initial conditions as (c). Find the solution of the initial value problem. You can use Matlab to solve the linear systems.

2. For the following matrices: Find a nonsingular matrix $V \in \mathbb{C}^{n\times n}$ and a matrix $B \in \mathbb{C}^{n\times n}$ in Jordan form (i.e., having Jordan boxes along diagonal) such that $AV = VB$. Do this by hand (hint: there are only two different eigenvalues, and one eigenvalue is easy to see). Note: Use $(A - \lambda I)\vec{w} = \vec{v}$ to find a generalized eigenvector $\vec{w}$; here the eigenvector $\vec{v}$ has to be carefully chosen so that a solution $\vec{w}$ exists.
In Matlab use `[V,D]=eig(A)` with symbolic matrices. Then use `[V,B]=jordan(A)` .

$$\text{(i) } A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 4 \\ 0 & -3 & -1 & -2 \\ 0 & 3 & -1 & 0 \end{bmatrix}, \qquad \text{(ii) } A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & -3 & -1 & -2 \\ 0 & 3 & -1 & 0 \end{bmatrix}$$