Practice problems for Exam #1

1. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 2 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$. You **must** use the result of (a) to answer (b)–(g).

I recommend that you check your result of (a) with the solution before continuing.

(a) Perform Gaussian elimination to find the row echelon form U, the matrix L of multipliers, and the permutation vector p. **Always use the first available pivot candidate.**

(b) Find a basis for $\operatorname{range} A$.

(c) Find a basis for $\operatorname{null} A$.

(d) Find a basis for $\operatorname{range} A^\top$.

(e) Find a basis for $\operatorname{null} A^\top$.

(f) Consider a linear system $Ax = b$ where $b \in \mathbb{R}^4$ is given. How many conditions does the vector b have to satisfy so that a solution exists? State these conditions (using your earlier results).

(g) Consider the linear system $Ax = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$. Find the general solution.

2. Assume we have a matrix $A \in \mathbb{R}^{2\times 4}$

(a) What are the possible values for $r = \operatorname{rank} A$? Give an example matrix A for each case!

(b) Assume $\operatorname{rank} A = 1$. What is the dimension of the spaces $\operatorname{range} A$, $\operatorname{null} A$, $\operatorname{range} A^\top$, $A^\top$?

3. For a matrix $A \in \mathbb{R}^{3\times 5}$ we obtain the row echelon form $U = \begin{bmatrix} 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and the permuation vector $p = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

DO NOT find the matrix A!

(a) Which of the columns of the matrix A form a basis for the column space?

(b) Which of the rows of the matrix A form a basis for the row space?

(c) Find a basis for the orthogonal complement of the row space.