Exam #2

You need to **show all your work** and **solve the problems in the specified way** in order to get credit.

1. (25 pts)

(a) (5 pts) Let $P = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -1 & 1 \\ -2 & -1 \end{bmatrix}$. Check that $P^\top P$ is a diagonal matrix. What does this mean for the columns of P?

(b) (20 pts) Let $A = P \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. DO **NOT** TRY TO FIND THE MATRIX A! Use this decomposition to find $c \in \mathbb{R}^2$ such that $\left\| Ac - \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} \right\|$ is minimal.

2. (25 pts) We are given the data values $\begin{array}{c|ccc} t_j & -1 & 0 & 1 \\ \hline y_j & 1 & 2 & 4 \end{array}$. Find the least squares fit with a function of the form $y = c_1 + c_2 t^2$ and find the **norm of the residual**.

3. (25 pts) Let $A = \begin{bmatrix} 4 & 0 & 0 \\ -3 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$.

(a) (10 pts) Use the expansion formula for the determinant to write down $\det(A - \lambda I)$. Then find the eigenvalues $\lambda_1, \lambda_2, \lambda_3$.

(b) (15 pts) Find a nonsingular matrix $V \in \mathbb{R}^{3\times 3}$ and a diagonal matrix B so that we have $AV = VB$.

4. (25 pts) The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & -2 \end{bmatrix}$ are 2 and -3. **Find the dimension of each eigenspace** (you don't need to find the eigenvectors!). Is the matrix diagonizable?