# Practice problems for Exam #2

**1.** We are given the data values $\begin{array}{c|cccc} t_j & -2 & -1 & 1 & 2 \\ \hline y_j & 3 & 1 & 2 & 4 \end{array}$. Find the least squares fit with a function of the form $y = c_1 t + c_2 |t|$.

**2.**

**(a)** For the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ find a decomposition $A = P \begin{bmatrix} 1 & s_{12} & s_{13} \\ 0 & 1 & s_{23} \\ 0 & 0 & 1 \end{bmatrix}$ where the columns of the matrix $P \in \mathbb{R}^{4\times 3}$ are orthogonal on each other.

**(b)** For a *different* matrix $A$ we obtain the decomposition $A = \underbrace{\begin{bmatrix} -1 & 2 \\ 2 & -1 \\ 2 & 2 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}}_{S}$ where the columns of the matrix $P$ are orthogonal on each other. Use this to find $c \in \mathbb{R}^2$ such that $\left\| Ac - \begin{bmatrix} 5 \\ 2 \\ 5 \end{bmatrix} \right\|$ is minimal. DO **NOT** TRY TO FIND THE MATRIX $A$!

**3.** Use the expansion formula to find $\det \begin{bmatrix} 0 & 2 & 3 & 5 \\ 2 & 7 & 8 & 9 \\ 0 & 2 & 3 & 1 \\ 0 & 4 & 0 & 0 \end{bmatrix}$. Hint: try to pick convenient rows or columns.

**4.** Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & -8 & 5 & 5 \\ 1 & -5 & 2 & 5 \\ 1 & -5 & 5 & 2 \end{bmatrix}$. DO **NOT** TRY TO FIND THE CHARACTERISTIC POLYNOMIAL!

**(a)** Write down the matrix $M = A - \lambda I$. Look at this matrix and try to guess a value $\lambda$ which makes $M$ singular (without using $\det M$). Find a basis for the eigenspace for this eigenvalue.

**(b)** Find a basis for the eigenspace for $\lambda = -3$.

**(c)** The matrix $A$ has only two different eigenvalues: the eigenvalue from (a), and $\lambda = -3$. Is the matrix $A$ diagonizable? Explain!

**5.** Find all eigenvalues and eigenvectors for the matrices

$$\text{(i)} \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}, \qquad \text{(ii)} \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}, \qquad \text{(iii)} \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}, \qquad \text{(iv)}\, A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Which of these matrices are diagonalizable?