Practice problems for final exam

Problems for Jordan normal form and quadratic forms

1. Let $A = \begin{bmatrix} 0 & 4 & 0 \\ -1 & -4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. Find the Jordan normal form B and the nonsingular matrix $V \in \mathbb{R}^{3\times 3}$ so that $AV = VB$.

2. The matrix $A = \begin{bmatrix} 10 & 5 & 1 \\ 2 & 9 & 5 \\ -2 & -3 & 5 \end{bmatrix}$ has only the eigenvalue $\lambda = 8$. Find the Jordan normal form B and the nonsingular matrix $V \in R^{3\times 3}$ so that $AV = VB$.

3. For the quadratic form $q(x_1, x_2) = 3x_1^2 - 4x_1x_2$ find the symmetric matrix $A \in \mathbb{R}^{2\times 2}$ such that $q(x) = x^\top Ax$. Find the eigenvalues. Is A positive semidefinite? If not, find $x \neq \vec{0}$ such that $x^\top Ax < 0$.

4. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and consider the quadratic form $q(x) = x^\top Ax$. Find a nonsingular matrix $V \in \mathbb{R}^{3\times 3}$ such that $B = V^\top AV$ is diagonal. Is A positive definite?
Hint: You should get $p(\lambda) = -\lambda^3 + 5\lambda^2 - 4\lambda$.

Problems for Gaussian elimination and row echelon form

1. Let $A = \begin{bmatrix} 0 & -4 & 4 \\ -3 & -1 & -2 \\ -1 & 1 & -2 \\ 0 & -4 & 4 \end{bmatrix}$. Use Gaussian elimination to find the row echelon form U and L, p. Always use the first available pivot candidate in each column. What are the **dimensions** of of $\mathrm{null}(A)$, $\mathrm{range}(A)$, $\mathrm{null}(A^\top)$, $\mathrm{range}(A^\top)$?

2. Let $A = \begin{bmatrix} 2 & 2 & -3 & 4 \\ 2 & 2 & -1 & 2 \\ 4 & 4 & 6 & -4 \end{bmatrix}$. Use Gaussian elimination to find the row echelon form U and L, p. Always use the first available pivot candidate in each column. What are the **dimensions** of of $\mathrm{null}(A)$, $\mathrm{range}(A)$, $\mathrm{null}(A^\top)$, $\mathrm{range}(A^\top)$?

3. For some matrix A we obtain from Gaussian elimination $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 3 & 1 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $p = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}$.

(a) Find a basis for $\mathrm{null}(A)$, $\mathrm{range}(A)$, $\mathrm{null}(A^\top)$, $\mathrm{range}(A^\top)$. Use the given L, U, p, do NOT try to find A or $A^\top$.

(b) Find the general solution of the linear system $Ax = \begin{bmatrix} -2 \\ 5 \\ 3 \\ -2 \end{bmatrix}$.