Assignment #1, due Thursday, February 5 at 10pm

Use Matlab for all computations. Use `format long g` in Matlab to see all computed digits.
Only print out numerical results which are asked for in the problem (put a semicolon at the end of your commands).
You **MUST** use a "Live Script" **`mlx` file**. Use text fields (not code comments) to answer all questions. Use **`clearvars`** as the first command in your script file.
DO NOT use symbolic Matlab commands like `sym`, `syms`, `taylor`, `subs`.
Please read "**How to hand in homeworks**" on the ELMS page.

1. **(i)** Find the Taylor polynomial $p_n(x) = f(x_0) + \cdots + f^{(n)}(x_0)(x - x_0)^n/n!$ **by hand**. Then evaluate $p_n(x)$ using Matlab.
   (ii) Find an upper bound $|f(x) - p_n(x)| \leq \cdots$ using the remainder term. Use Matlab to evaluate the upper bound and $f(x) - p_n(x)$.

   (a) $y = 2.02^{-1}$ with $x_0 = 2$, $n = 5$

   (b) $y = \cos 0.2$ with $x_0 = 0$, $n = 5$

   (c) $y = \ln(1.03)$ with $x_0 = 1$, $n = 2$

2. We want to approximate $z = \sqrt{a}$ for $a = 4.2$.

   (a) Method 1: Use the Taylor approximation $y = p_2(x)$ and $x_0 = 4$. Find an error bound $|y - z| \leq \cdots$.

   (b) Method 2: Use the "**Babylonian algorithm**":
   We start with an **initial guess** y. Then $z = \sqrt{a}$ is between y and a/y, and we use the **midpoint**

   $$\boxed{y_{\text{new}} := \tfrac{1}{2}\left(y + \frac{a}{y}\right)}$$

   as the new guess. Then we repeat this.
   Explain why we have the **error bound** $|y_{\text{new}} - z| \leq E$ with $E = \frac{1}{2}\left|y - \frac{a}{y}\right|$.
   Write Matlab code which starts with the initial guess $y = 2$ and prints **prints after each iteration** y_{new}, E, E/E_{old}^2 where E_{old} is the error bound from the previous iteration, using
   `fprintf('ynew=%-17.16g, E=%.3e, ratio=%g\n',ynew,E,E/Eold^2)`
   Do not print anything else.
   Iterate until $E \leq 10^{-14}$. What do the ratios E/E_{old}^2 tell us?

3. We use **single precision**. For the following examples (a), (b), (c) do the following:
   (i) Find an expression for the **condition number** $\boxed{c_f(x) = \dfrac{x \cdot f'(x)}{f(x)}}$ **using pencil and paper**. Evaluate $c_f(x)$ for the given value of x in Matlab and find the **unavoidable error** $\boxed{|c_f(x)|\,\varepsilon_M + \varepsilon_M}$. Here $\varepsilon_M \approx 10^{-7}$ is the machine accuracy.
   (ii) Evaluate the formula for y in Matlab using **single precision**, yielding `y`. **You have to use `single(···)` for ALL input values.**
   Then **evaluate the formula** in **double precision**, yielding `yd` (see example on ELMS page).
   Then compute the **actual relative error** of `y` with `relerr=(double(y)-yd)/yd` .
   Compare this relative error with the unavoidable error. **Was this computation numerically stable?**

   (a) For $x = 10^{-4}$ compute $y = \sqrt{16 - x} - 4$.

   (b) For $x = 10^{-4}$ compute $y = \ln(1 - x)$.

   (c) For $x = 1.2 \cdot 10^{-4}$ compute $y = \sin(1/x)$.