Practice Problems for Exam 2

1. Consider the function $f(x) = \sin(\frac{\pi}{2}x)$

(a) Find the interpolating polynomial $p(x)$ for the nodes $x_1 = 0$, $x_2 = 1$, $x_3 = 2$.

(b) Use the error formula to give an upper bound for $|f(x) - p(x)|$ with $x \in [-\frac{1}{2}, \frac{5}{2}]$.

(c) For the points x_1, x_2, x_3 find the best least squares fit with a function of the form $g(x) = c_1 + c_2 x$.

2. Consider the nonlinear system

$$\begin{aligned} x_1 + x_1x_2 + x_2 &= 2 \\ x_1 - x_2 - x_1x_2^2 &= 0 \end{aligned}$$

Perform one step of the Newton method starting with $(1, 1)$.

3. Consider the initial value problem

$$y'' + y' + y = t, \qquad y(1) = 1, \quad y'(1) = 2$$

(a) Perform one step of the *Euler method* with $h = 1$ and give the resulting approximation for $y(2)$.

(b) Perform one step of the *improved Euler method* with $h = 1$ and give the resulting approximation for $y(2)$.

(c) Perform one step of the *backward Euler method* with $h = 1$ and give the resulting approximation for $y(2)$.

(d) Write a Matlab function `z=IVP(a,b)` using `ode45` which solves the initial value problem with $y(1) = a$, $y'(1) = b$ and returns an approximation z for $y(2)$.