Practice problems for Exam 2

1. We are given the following information about $f(x)$:

$$f(1) = 2, \quad f'(1) = 1, \quad f(3) = 2, \quad f'(3) = -1$$

(a) Write down the divided difference table and the interpolating polynomial in Newton form.

(b) Assume we know that the 4th derivative satisfies $\left|f^{(4)}(x)\right| \leq 10$ for $x \in [1, 2]$. Find an upper bound for $|f(1.5) - p(1.5)|$.

2. We want to find x such that $x + x^5 = 3$.

(a) Perform one step of the bisection method with $a_0 = 1$, $b_0 = 2$. Find k such that $|b_k - a_k| \leq 10^{-6}$.

(b) Perform one step of the secant method with $x_0 = 1$, $x_1 = 2$ to find x_2.

(c) Will the Newton method converge if we start with x_0 sufficiently close to the solution x_*? Explain.

3. Consider the nonlinear system

$$x_1 + x_1x_2 + x_2 = 2, \qquad x_1 - x_2 - x_1x_2^2 = 0$$

Perform one step of the Newton method starting with initial guess $x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

4. Consider the nonlinear system

$$x_1 = \frac{1 + x_2 + \cos(x_1 + x_2)}{4}$$
$$x_2 = \frac{1 + x_1 + \sin(x_1 - x_2)}{4}$$

(a) Show that this nonlinear system has a unique solution in the square $[0, 3] \times [0, 3]$. *Hint:* Use the ∞-norm.

(b) Starting with $x^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ find $x^{(1)}$ using the fixed point iteration. What can you conclude about the location of the solution x^*? Give the answer in the form $x^* \in [a_1, b_1] \times [a_2, b_2]$.
Hint: Use the a-posteriori estimate $\left\|x^{(k)} - x^*\right\| \leq \frac{q}{1-q} \left\|x^{(k)} - x^{(k-1)}\right\|$.