Practice Problem for Numerical Integration

1. Consider the integral $I = \int_0^2 x^3 dx$.

   (a) Find the value of the **(i)** midpoint rule, **(ii)** trapezoid rule, **(iii)** Simpson rule (on the whole interval). Find an upper bound for the error $|Q - I| \leq \cdots$ using the error formulas in each case.

   (b) Find the value of the composite trapezoid rule Q_2^{Trap} with 2 subintervals of equal size.

   (c) Find a value N such that we can guarantee $\left| Q_N^{\text{Trap}} - I \right| \leq 10^{-10}$ for the composite trapezoid rule with N intervals of equal size.