MATH 141 – CALCULUS II
THIRD MIDTERM EXAM SOLUTIONS

(1) Use the substitution $x = 3\sec\theta$, $dx = 3\sec\theta\tan\theta d\theta$. Then

$$\begin{aligned}\int_{3\sqrt{2}}^{6} \frac{dx}{x^2\sqrt{x^2-9}} &= \int_{\pi/4}^{\pi/3} \frac{3\sec\theta\tan\theta\, d\theta}{9\sec^2\theta\cdot 3\tan\theta}\\ &= \frac{1}{9}\int_{\pi/4}^{\pi/3} \cos\theta\, d\theta\\ &= \frac{1}{9}\sin\theta\Big]_{\pi/4}^{\pi/3}\\ &= \frac{\sqrt{3}-\sqrt{2}}{18}\end{aligned}$$

(2) For (a)

$$\frac{x^3+x+1}{x-1} = x^2+x+2+\frac{3}{x-1}$$
$$\int \frac{x^3+x+1}{x-1}\,dx = x^3/3 + x^2/2 + 2x + 3\ln|x-1| + C$$

For (b)

$$\frac{x+1}{x^3-2x^2+x} = \frac{1}{x} - \frac{1}{x-1} + \frac{2}{(x-1)^2}$$
$$\int \frac{x+1}{x^3-2x^2+x}\,dx = \ln|x| - \ln|x-1| - \frac{2}{x-1} + C$$

(3) For (a), evaluate the indefinite integral: $F(x) = \frac{1}{2}e^x(\sin x - \cos x)$. This does not have a limit as $x \to \infty$. Indeed, for values $n\pi$ where n is an integer, $F(n\pi) \to +\infty$ for n odd and $F(n\pi) \to -\infty$ for n even. For (b), evaluate directly

$$\int_2^\infty \frac{dx}{x(\ln x)^3}\,dx = \lim_{b\to\infty} \frac{-1}{2(\ln x)^2}\Big]_2^b = \frac{1}{2(\ln 2)^2}$$

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(4) (a) Divide numerator and denomenator by n^2: the limit is $-5/3$. For (b)

$$\begin{aligned}
\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-n\right) &= \lim_{n\to\infty}\left(\sqrt{n^2+n+1}-n\right)\cdot\frac{\sqrt{n^2+n+1}+n}{\sqrt{n^2+n+1}+n} \\
&= \lim_{n\to\infty}\frac{n^2+n+1-n^2}{\sqrt{n^2+n+1}+n} \\
&= \lim_{n\to\infty}\frac{1+1/n}{\sqrt{1+1/n+1/n^2}+1} \\
&= \frac{1}{2}
\end{aligned}$$