MATH 241 – CALCULUS III
FIRST MIDTERM EXAM

Instructions. Answer each question on a separate answer sheet. Show all your work. Be sure your name, section number, and problem number are on each answer sheet, and that you have copied and signed the honor pledge on the first answer sheet. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) (a) Find the unit vector in the direction of $\mathbf{u} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

(b) Compute the projection $\text{pr}_{\mathbf{b}}\mathbf{a}$ of the vector $\mathbf{a} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ on the vector $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

(c) Find the equation of the plane passing through the points $(0, 0, 0)$, $(1, 1, -1)$, and $(2, 1, 1)$.

(2) (a) Find $\dfrac{d}{dt}(\mathbf{F} \cdot (\mathbf{G} \times \mathbf{H}))$ at $t = 0$, if

$$\mathbf{F}(0) = \hat{\mathbf{i}}\ , \mathbf{F}'(0) = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$
$$\mathbf{G}(0) = \hat{\mathbf{j}}\ , \mathbf{G}'(0) = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$
$$\mathbf{H}(0) = \hat{\mathbf{k}}\ , \mathbf{H}'(0) = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

(b) Find $\dfrac{d}{dt}\mathbf{F}(g(t))$ at $t = 0$, where $\mathbf{F}(t) = \cos t\,\hat{\mathbf{i}} + \sin t\,\hat{\mathbf{j}} + t\,\hat{\mathbf{k}}$, and $g(t) = \pi e^{2t}$.

(c) Let

$$\mathbf{G}(t) = \int_0^t \mathbf{F}(x)dx$$

where $\mathbf{F}$ is as in part (b). What is $\mathbf{G}'(\pi)$?

(3) (a) What is the tangent vector $\mathbf{T}(t)$ to the parametrized curve given by

$$\mathbf{r}(t) = 2t\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}} + \ln t\,\hat{\mathbf{k}}$$

(b) Find the length of $\mathbf{r}(t)$ for $1 \le t \le 2$.

(c) What is ds/dt, where s is the arc length parameter?

(4) Compute the velocity, speed, and acceleration of the particle whose position is given by

$$\mathbf{r}(t) = 3t\,\hat{\mathbf{i}} + t\,\hat{\mathbf{j}} - t^2\,\hat{\mathbf{k}}$$

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