MATH 241 – CALCULUS III
FIRST MIDTERM EXAM SOLUTIONS

(1) (a) $(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})/\sqrt{3}$.

   (b) $(2/3)(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$.

   (c) $2x - 3y - z = 0$

(2) (a) Apply the values to $\mathbf{F}' \cdot (\mathbf{G} \times \mathbf{H}) + \mathbf{F} \cdot (\mathbf{G}' \times \mathbf{H}) + \mathbf{F} \cdot (\mathbf{G} \times \mathbf{H}')$. The answer is 0.
   (b) $\mathbf{F}' = -\sin t\, \hat{\mathbf{i}} + \cos t\, \hat{\mathbf{j}} + \hat{\mathbf{k}}$; at $t = \pi$ this is $-\hat{\mathbf{j}} + \hat{\mathbf{k}}$. Since $g'(0) = 2\pi$, the result is $2\pi(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$.
   (c) $\mathbf{G}'(\pi) = \mathbf{F}(\pi) = -\hat{\mathbf{i}} + \pi\,\hat{\mathbf{k}}$.

(3) (a) $\mathbf{T} = \dfrac{\mathbf{r}'}{\|\mathbf{r}'\|} = \dfrac{2\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}} + t^2\,\hat{\mathbf{k}}}{(4 + 4t^2 + 1/t^2)^{1/2}}$.

   (b)

$$\begin{aligned}
\text{Length} &= \int_1^2 (4 + 4t^2 + 1/t^2)^{1/2}dt \\
&= \int_1^2 (4t^4 + 4t^2 + 1)^{1/2}\frac{dt}{t} \\
&= \int_1^2 (2t + 1/t)dt = 3 + \ln 2
\end{aligned}$$

   (c) $ds/dt = \|\mathbf{r}'\| = (4 + 4t^2 + 1/t^2)^{1/2}$.

(4)

$$\begin{aligned}
\mathbf{v} = \mathbf{r}' &= 3\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2t\,\hat{\mathbf{k}} \\
\mathbf{a} = \mathbf{r}'' &= -2\,\hat{\mathbf{k}} \\
\text{Speed} = \|\mathbf{v}\| &= \sqrt{10 + 4t^2}
\end{aligned}$$