MATH 241 – CALCULUS III
SECOND MIDTERM EXAM

Instructions. Answer each question on a separate answer sheet. Show all your work. Be sure your name, section number, and problem number are on each answer sheet, and that you have copied and signed the honor pledge on the first answer sheet. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) (a) Let $f(x, y) = ye^{3x}$. For which unit vector $\mathbf{u}$ is $D_{\mathbf{u}}f(0, 1)$ the greatest?
(b) Compute the equation of the tangent plane at the point $(0, 0, 1)$ to the surface given by the equation

$$3e^x + 2xz + \sin(x + 2y) = 3z$$

(2) (a) Determine if the limit

$$\lim_{(x,y)\to(0,0)} \frac{(x + y)^2}{x^2 + y^2}$$

exists, and if it does compute it. Justify your answer.
(b) Near the point $(0, 0, 1)$, the equation of the surface in problem 1(b) determines x as a function of (y, z). Compute $\partial x/\partial z$ when $(x, y, z) = (0, 0, 1)$.

(3) (a) Suppose $\mathbf{r}(s)$ is a curve in 3-space parametrized by arc length with tangent vector $\mathbf{T}(s)$. How is the normal vector $\mathbf{N}(s)$ defined, and why is it perpendicular to $\mathbf{T}(s)$ for every s?
(b) The torsion $\tau(s)$ is defined by $d\mathbf{B}(s)/ds = -\tau(s)\mathbf{N}(s)$, where $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ is the binormal. Show that

$$\mathbf{B}(s) \cdot \frac{d\mathbf{N}}{ds}(s) = \tau(s)$$

for every s.

(4) Find and classify the critical points of $f(x, y) = 3x^2 - 3xy^2 + y^3 + 3y^2$.

Date: Oct 25, 2010.