# MATH 241 – CALCULUS III
# FOURTH MIDTERM EXAM

**Instructions.** Answer each question on a separate answer sheet. Show all your work. Be sure your name, section number, and problem number are on each answer sheet, and that you have copied and signed the honor pledge on the first answer sheet. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) Evaluate the following line integrals.

(a) $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = xe^y\,\hat{\mathbf{i}} + \hat{\mathbf{j}}$, and $C$ is the portion of the graph of $y = x^2$ from the points $(0, 0)$ to $(1, 1)$.

(b) $\int_C (\cos^3 x + e^x)dx + e^{y^2}dy$, where $C$ is the closed curve $x^6 + y^8 = 1$ oriented counterclockwise.

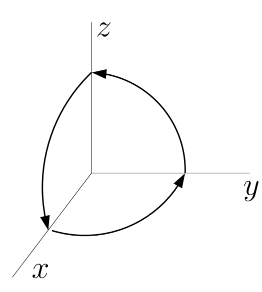
(c) $\int_C (2xy^2 + 1)dx + (2x^2y)dy$, where $C$ is any curve from $(-1, 2)$ to $(2, 3)$.

(2) Let $S$ be the surface $x^2 + (y/2)^2 + (z/3)^2 = 1$, $z \geq 0$. Compute the surface integral $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\,dS$, where $\mathbf{F} = y\,\hat{\mathbf{i}} - 4x\,\hat{\mathbf{j}} + z^2\,\hat{\mathbf{k}}$.

(3) (a) State Stokes' Theorem.

(b) Compute $\nabla \times \mathbf{F}$, where $\mathbf{F} = z(x + 1)\,\hat{\mathbf{i}} + (zy - x)\,\hat{\mathbf{j}} + x\,\hat{\mathbf{k}}$.

(c) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the union of the curves given by the intersection of the unit sphere $x^2 + y^2 + z^2 = 1$ with the coordinate planes in the first octant. It has the orientation indicated in the figure below.




(4) (a) State the Divergence Theorem.

(b) Compute $\nabla \cdot \mathbf{F}$, where $\mathbf{F} = xy\,\hat{\mathbf{i}} - y^2\,\hat{\mathbf{j}} - \sin(-z^2y)\,\hat{\mathbf{k}}$.

(c) Compute $\nabla \cdot (\nabla \times \mathbf{F})$.

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*Date*: Dec 8, 2010.