Math 246H – Exam #1

(1) Solve the initial value problems for $y = y(t)$ and state the maximal interval of existence for each solution (20 pts each).

(a) $ty' - 2y = 3t^2 \ln t \ ; \ y(1) = 2.$

(b) $y' = \dfrac{2x}{y\sqrt{x^2 - 9}} \ ; \ y(5) = -1.$

(2) Find the equilibrium solutions and discuss their stability for the equation:

$$y' = y(9 - y^2)(y - 2)$$

Make a plot of the integral curves to illustrate your result (20 pts).

(3) Solve the initial value problem (20 pts):

$$2x + y^2 + 2(x + 1)yy' = 0 \ ; \ y(0) = 2$$

(4) A college graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k, determine the payment rate k that is required to pay off the loan in 3 years. Also, for this value of k determine how much interest is paid during the 3-year period. (20 pts).