Math 246H – Exam #2

(1) (15 pts) Find the *general solution* to the equation: $y''' - 3y'' + 3y' - y = 0$.

(2) (15 pts) Notice that $y_1(t) = e^t$ is a solution to the equation: $(t-1)y'' - ty' + y = 0$. Find another solution independent of y_1.

(3) (20 pts) Find the solution of the following initial value problem.

$$y'' + 2y' + 2y = 0 \ , \ \ y(\pi/4) = 1 \ , \ \ y'(\pi/4) = 3$$

(4) (30 pts) Suppose that $y_p(t)$ is a *particular* solution to the equation

$$(*) \qquad y'' - 5y' + 6y = g(t)$$

for some function $g(t)$.

(a) Write the *general solution* to $(*)$ in terms of y_p.

(b) Use variation of parameters to find a particular solution to $(*)$. Express your result in terms of integrals involving g.

(5) (10 pts) Compute the Wronskian of the functions $\{1, e^{2t}, e^{-t}\}$.

(6) (10 pts) Compute the Wronskian of $\{y_1, y_2, y_3\}$ at $t = 1$, where y_1, y_2, y_3 are solutions to $y''' + 2y'' + ty' + (\sin t)y = 0$, satisfying the initial conditions

$$\begin{array}{lll} y_1(0) = 1 & y_2(0) = 3 & y_3(0) = 2 \\ y_1'(0) = 0 & y_2'(0) = -1 & y_3'(0) = 5 \\ y_1''(0) = 0 & y_2''(0) = 0 & y_3''(0) = 2 \end{array}$$